A PRACTICAL SYSTEM

OF

MENSURATION

OF

SUPERFICIES AND SOLIDS:

DESIGNED ESPECIALLY FOR

ADVANCED SCHOLARS IN SCHOOLS AND ACADEMIES.

BY REV. J. M. SCRIBNER, A. M.
LATE PRINCIPAL OF THE AUBURN FEMALE SEMINARY.

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RECOMMENDATIONS.

I have examined, with much care, the proof sheets of "*Mensuration of Superficies and Solids*," by the Rev. J. M. Scribner, and it gives me pleasure to state that I have been very favorably impressed with the character of the book. I am satisfied that it is well adapted to the wants and capacities of those for whom it is intended; more so than any other production, of a similar kind, with which I am acquainted.

Wm. HOPKINS,
Principal of the Auburn Academy.

Auburn, April 19th, 1844.

I fully concur in the foregoing opinion of Mr. Hopkins, and will add, that I hope to see the work introduced as a Text Book in our Schools.

P. H. PERRY,
Town Superintendeut of Com. Schools.

Auburn, April 20, 1844.

Dear Sir: Your "*Mensuration of Superficies and Solids*," is a work, the want of which has long been felt in our Common Schools. Its purely practical character must at once commend it to the favor of the discerning; and our older pupils *should* not, and I trust will not, rest satisfied short of an acquaintance with the principles which the work so clearly illustrates and applies.

Very Respectfully, Yours &c.

E. G. STORKE,
County Superintendent of Common Schools.

Rev. J. M. Scribner.

PREFACE.

Mathematics is studied either as a necessary branch of a finished education, or by those who are anxious to store their minds with scientific facts and principles, to qualify them for active usefulness and the various employments and professions to which they may be called in after life. The scholar by studying a system of theory may have all his ends answered, but the practical mechanic, the engineer, and the man of business cannot follow their professions and perform their part with propriety without being expert in most branches of mathematics.

The work now presented to the public, had its origin in a desire on the part of the author to draw up a practical treatise on the *Mensuration of Superficies and Solids* for the use of advanced scholars in Schools and Academies. It is conceded on all hands that this important and useful branch of mathematics has been too much neglected, and that the time has come when a work comprised within moderate limits, and adapted to the wants of our young men of ardour and enterprise, who intend devoting them-

selves to mechanical pursuits, can no longer be dispensed with in our institutions of elementary education. And it is believed, also, that no production of the kind, well calculated and adapted to the wants and capacities of those acquiring an elementary education, has as yet been offered to the public.

Lord Brougham, in his "Practical Observations upon the education of the People," very judiciously remarks, that "a most essential service will be rendered to the cause of knowledge by him who shall devote his time to the composition of elementary treatises on mathematics, sufficiently clear, and yet sufficiently compendious to exemplify the method of reasoning employed in that science, and to impart an accurate knowledge of the most useful fundamental propositions with their application to practical purposes." I do not flatter myself that this brief compend will be thought fully adequate to supply even in this department, all the desiderata to which the above writer alludes, yet I could but be gratified after the work was nearly completed, to find that the views which guided me in its execution harmonized with the opinions of others distinguished as mathematicians and experienced as teachers. The author has aimed to present this treatise in the most condensed form which the nature and importance of the work would admit, well knowing that it is a great encouragement to the scholar to proceed when the end of the task is in view, and that

nothing is more discouraging to a beginner, than to be told that the branch of science he is about to learn extends to a great distance. The apparent length of the labor sets proficiency at so distant a view, that their limited time seems altogether too short to accomplish the desired object, and if the natural desire and thirst for knowledge be thus nipped in the bud by such an unneccessary view of the subject, it will be exceedingly difficult afterwards to make any one apply diligently and cheerfully to the study.

As a school book, it is adapted to lead the mind, and to encourage by rules given in terms which it is believed the scholar can easily comprehend; and even those who have not had an opportunity of devoting their time to the study of mathematics will, it is presumed, with the aid the following pages will afford them, be able to perform their part with propriety in this branch of mathematics.

As my design in the publication of this work was not *originality of materials*, but rather a plan, arrangement and execution adapted to general use, I have freely consulted the valuable works of Day, Hutton, Gregory and Legendre, authors whose works *ought* to be examined in the preparation of a compend like this; and I freely acknowledge my indebtedness to their labors, for many valuable ideas. I am aware of the great difficulty of preventing errors from creeping into a work of this kind; and should it be found

to contain imperfections, those who may make the discoveries are respectfully solicited to forward any communications or amendments, to the author, which to them may appear necessary to make the work complete. Should a second edition of the work be called for, the author will furnish a Section on Cask Gauging.

J. M. SCRIBNER.

Auburn, April, 1844.

CONTENTS.

UNITED STATES WEIGHTS AND MEASURES.

LONG MEASURE.

For Measuring Length without regard to Breadth.

3 barley corns make	1 inch.
12 inches	1 foot.
6 feet	1 fathom.
3 feet	1 yard.
5½ yards or 16½ feet	1 rod or pole.
40 rods or 220 yards	1 furlong.
8 furlongs make	1 mile.
3 miles	1 league.
69½ statute miles	1 degree.
60 geographic miles	1 degree.
360 degrees	1 great circle of the earth's circumference.

CLOTH MEASURE.

This Measure is used for measuring Goods sold by the Yard, Ell, &c.

2¼ inches make	1 nail.
4 nails	1 quarter.
4 quarters	1 yard.
3 quarters make	1 Flemish ell.
5 quarters	1 English ell.
6 quarters	1 French ell.

SQUARE MEASURE.

144	square inches	make	1	square foot.
9	" feet	"	1	" yard.
272¼	" feet	"	1	" rod, perch, or pole.
40	" rods	"	1	" rood.
4	" roods	"	1	" acre.
640	" acres	"	1	square mile.

NOTE.—By this measure land and Artificers' work are computed; also boards, pavements, plastering, painting and every dimension of length and breadth only.

CUBIC OR SOLID MEASURE.

1728	cubic inches	make	1 cubic foot.
46656	" "	"	27 cubic feet.
27	" feet	"	1 cubic yard.
50	" " of round timber	make	1 ton.
40	" " of hewn "	"	1 ton.
42	" " of shipping "	"	1 ton.
16	" "	"	1 cord foot.
8 cord feet or 128 cubic feet		"	1 cord of wood.

MISCELLANEOUS.

1 chaldron=36 bushels or	57.25 cubic feet.
Dry gallon of New York	276.48 cubic inches.
1 perch of stone	24.75 cubic feet.

MEASURES OF CAPACITY.

Wine Measure.

4 gills make	1 pint.	42 gallons make	1 tierce.
2 pints	1 quart.	63 "	1 hogshead.
4 quarts	1 gallon.	2 hogsheads	1 pipe or butt.
$31\frac{1}{2}$ gallons	1 barrel.	2 pipes	1 tun.

ALE AND BEER MEASURE.

Ale.

2 pints make	1 quart.	2 kilderkins make	1 barrel.
4 quarts	1 gallon.	36 gallons	1 barrel.
9 gallons	1 ferkin.	54 "	1 hogshead.
2 ferkins	1 kilderkin.	2 hogsheads	1 butt.

Beer.

$35\frac{1}{4}$ cubic inches	=	1 pint.
$70\frac{1}{2}$ " "	=	1 quart.
282 " "	=	1 gallon.

NOTE.—The dry gallon contains $268\frac{4}{5}$ cubic inches; The wine gallon contains 231 cubic inches; The beer gallon contains 282 cubic inches; Hence 14553 cubic inches = 1 hogshead of wine, and 15228 cubic inches = 1 hogshead of Beer. The same standards continued in use in Great Britain as late as the year 1826, when the Act of Parliament for "Uniformity of Weights and Measures" came into operation, by which the Imperial gallon of 277.274 cubic inches was substituted for the *dry*, *beer*, and wine gallon.

AVOIRDUPOIS WEIGHT.

By this are weighed hay, grain, groceries and all coarse articles.

16 drams make	1 ounce.	4 quarters make	1 cwt.
16 ounces	1 pound.	20 cwt.	1 ton.
28 pounds	1 quarter.	1 lb. = 14 oz 11 dwt. 16 gr. troy.	

7000 troy grains	=	1 lb. avoirdupois.
5760 " "	=	1 lb. troy.
175 " pounds	=	144 lbs. avoirdupois.
175 " ounces	=	192 oz. "
$437\frac{1}{2}$ " grains	=	1 oz. "
1 " pound	=	.8228 lb. "

Formerly 28 pounds were reckoned for a quarter, making 112 pounds to the hundred, but the law and practice have made it nearly obsolete.

TROY WEIGHT.

By this weight are weighed Gold, Silver, Jewelry and all Liquors.

24 grains	make	1	pennyweight.
20 pennyweights		1	ounce.
12 ounces		1	pound.

NOTE.—An ounce of Gold is divided into 24 equal parts called carats, and an ounce of silver into 20 parts called pennyweights; therefore to distinguish fineness of metals, such Gold as will abide the fire without loss, is accounted 24 carats fine. If it lose 2 carats in trial it is called 22 carats fine, &c. A pound of Silver which loses nothing in trial is 12 ounces fine; but if it lose 3 pennyweights it is 11 ounces 17 pennyweights fine, &c.

DRY MEASURE.

For dry goods, as fruit, grain, seeds, &c.

2 pints	make	1 quart.
4 quarts		1 gallon.
8 quarts		1 peck.
4 pecks		1 bushel.
36 heaped bush.	make	1 chaldron of coals.
8 bushels		1 quarter.
5 quarters		1 load.

ENGLISH DRY OR CORN MEASURE.

Cubic Inches.		Pint.		Gallon.		Peck.		
34½	=	1						
272¼	=	8	=	1				
544½	=	16	=	2	=	1		
2178	=	64	=	8	=	4	=	1 Winchester bushel.

NOTE.—A heaped bushel is nearly ⅓ more. The Winchester bushel (so called because the standard measure was kept at Winchester) is 18½ inches diameter, and 8 inches deep. But if the corn gallon contains only 268.8 cubic inches, the measure will be as follows:

Cubic Inches.		Pint.		Quart.		Gallon.		Peck.		
33.6	=	1								
67.2	=	2	=	1						
268.8	=	8	=	4	=	1				
537.6	=	16	=	8	=	2	=	1		
2150.42	=	64	=	32	=	8	=	4	=	1 Winchester bushel.

NOTE.—The statute bushel is the *Winchester*, and contains 2150.42 cubic inches; but the number is supposed to vary in different states. In Connecticut it is 2198 which is 47.58 cubic inches larger than the Winchester bushel. The statute bushel of the State of New York contains 2211.84 cubic inches or 80 lbs. of pure water at a maximum density. A proposition has recently been made to the states by Congress for an uniform standard of Weights and Measures, but it has not yet been generally adopted.

The *Imperial* measure of capacity for coals, lime, potatoes, corn in the ear, fruit, and other goods commonly sold by *heaped measure*, is of the following dimensions.

2 gallons = 1 peck = 704 } cubic inches nearly.
8 " = 1 bushel = 2815½ }

The cone formed above the rim of the bushel should not be less than 6 inches. The outside diameter of the measure used for heaped goods, is to be, at least, double the depth; and consequently not less than the following dimensions:

DIMENSIONS OF THE BUSHEL.

19½ inches = outside diameter.
18½ " = inside "
8 " = depth.

Or, to have the bushel contain the same amount *even* with the rim, let its dimensions be 19½ inches *inside* diameter, and 9.43 inches deep.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream = 480 sheets.

SIZES OF DRAWING PAPER.

Wove Antique	52 by 31 inches		Imperial	29 by 21¼ inches.	
Uncle Sam	48 by 120 "		Super Royal	27 by 19 "	
Double Elephant	40 by 26 "		Royal	24 by 19 "	
Emperor	40 by 60 "		Medium	22 by 18 "	
Atlas	32 by 26 "		Demy	19 by 15½ "	
Colombier	33¾ by 23 "		Cap	13 by 16 "	
Elephant	27¾ by 23¼ "				

RELATIVE MINT VALUE OF FOREIGN GOLD COINS.

By Law of Congress, August, 1834.

Coins.		Weight. Pwt.	Gr.	Value.
BRAZIL,	1 Johannes	18		$17.068
"	1 Dobraon	34	12	32.714
"	1 Dobra	18	06	17.305
"	1 Moidore	6	22	6.560
"	1 Crusado	0	16¼	.638
ENGLAND,	1 Guinea	5	9¼	5.116
"	1 Sovereign	5	3¼	4.875
FRANCE,	1 Double Louis (1786)	10	11	9.694
"	1 Double Napoleon	8	7	7.713
COLUMBIA,	1 Doubloon	17	8½	15.538
MEXICO,	1 Doubloon	17	8½	15 538
PORTUGAL,	1 Doubloon	34	12	32.714
"	1 Dobra	18	6	17 305
"	1 Johannes	18		17.068
"	1 Moidore	6	22	6.560
"	1 Milrea		19¾	.780
SPAIN,	1 Doubloon (1772)	17	8½	16.030
"	1 Doubloon (1801)	17	9	15.538
"	1 Pistole	4	3½	3.883

MINT VALUE OF FOREIGN COINS.

England,	1 Shilling	= $0.244
France,	5 Francs	= 0.935
"	1 Sous	= 0.0093
Austria,	1 Crown, or rix dollar	= 0.97
"	1 Ducat	= 2.22
Prussia.	1 Ducat	= 2.202
Russia,	1 Ducat = 10 rubles	= 7.724
"	1 Ruble	= 0.748
Sweden,	1 Ducat	= 2.19
"	1 Rix dollar	= 1.08

AMERICAN STANDARD OF MONEY.

GOLD.

	Weight. Pwt.	Gr.
Eagle valued at $10	11	6
Half Eagle valued at $5	5	15
Quarter Eagle valued at $2,50	2	19½

23.2 grains of pure Gold = $1.00
United States Eagle till 1834 = $10.668

SILVER.

Dollar	17	7
Half Dollar	8	16
Quarter Dollar	4	4
French crown at one dollar eighteen cents	18	17

EXPLANATION OF CHARACTERS USED IN THIS WORK.

There are various characters or marks used in Arithmetic, to denote several of the operations and propositions, the chief of which are as follows:

$=$ *Equal*, ...	The sign of Equality; as, 100 cents $=$ \$1, signifies that 100 cents are equal to one dollar.
$-$ *Minus*, or *Less*,	The sign of Subtraction; as, $8-2=6$: that is, 8, less 2, is equal to 6.
$+$ *Plus*, or *More*,	The sign of Addition; as, $4+5=9$: that is, 4, added to 5, is equal to 9.
$\times$ *Multiply by*, . .	The sign of Multiplication; as, $6\times6=36$: that is, 6, multiplied by 6, is equal to 36.
$\div$ *Divided by*, . . .	The sign of Division; as, $12\div3=4$: that is, 12, divided by 3 is equal to 4.
: is to, : : so is, : to	The signs of proportion; as, $2 : 4 :: 8 : 16$: that is, as 2 is to 4, so is 8 to 16.
$7-2+5=10$	Shows that the difference between 7 and 2, added to 5, is equal to 10.
3^2	Signifies that the square of 3 is required; as, $3\times3=9$.
4^3	Signifies that the cube or third power of 4 is required; as, $4\times4\times4=64$.
$\sqrt{}$	Prefixed to any number, signifies that the square root of that number is required; as, $\sqrt{9}=3$.
$\sqrt[3]{}$	Prefixed to any number, signifies that the cube root of that number is required; as, $\sqrt[3]{64}=4$.

MENSURATION

OF

SUPERFICIES AND SOLIDS.

SECTION I.

MENSURATION OF SURFACES.

ART. 1. MENSURATION is that branch of Mathematics by which we ascertain the contents of superficial areas; the extension, solidities and capacities of bodies; and the lengths, breadths, &c. of various figures, either collectively or abstractly.

The Mensuration of Solids is divided into two parts:

I. Mensuration of the surfaces of solids;

II. The mensuration of their solidities.

In order to form correct estimates of the extent of surfaces and solids, various rules have been adopted, most of which, the most valuable and useful in practice, will be found accompanying their respective problems in the following work.

The surfaces or capacities of regular solids are readily calculated, but more intricacy attends the calculations of the surfaces and solid contents of many irregular bodies, as of frustrums, of pyramids, cones, &c.

With the following treatise before him, the student or the mechanic may speedily perform all the calculations that ordinarily occur in the practical details of his business.

Although Mensuration involves a knowledge of the elements of Geometry, yet it is not the object of this work to treat of that science at large. We shall therefore confine our exercises in this treatise to those measurements which will be most likely to prove beneficial to various classes of society, and especially to the operative mechanic, in the ordinary details of his business.

DEFINITIONS.

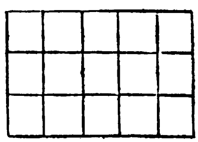
ART. 2. The following definitions, which are similar in substance to those found in Euclid, are here inserted for the convenience of reference, and to assist those who may be ignorant of Geometry in acquiring some knowledge of that science.

I. *Four-sided* figures are variously named, according to their relative position and length of their sides.

1

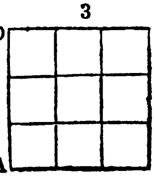
1. A *Parallelogram* has its opposite sides parallel and equal; as, *ABCD*, (fig. 1.)

2

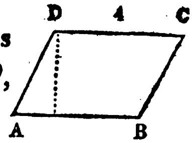


2. A *Rectangle*, or *Right Parallelogram* has its opposite sides equal, and all its angles right angles; as *ABCD*, (fig. 2.)

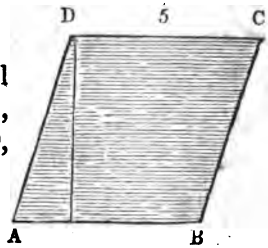
3



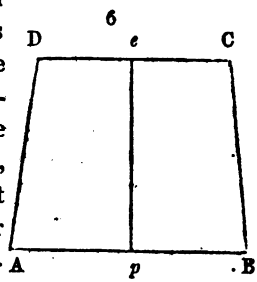
3. A *Square* is a figure whose sides are of equal length, and all its angles right angles; as, *ABCD*, (fig. 3.)



4. A *Rhomboid* has its opposite sides equal, and its angles oblique; as, *ABCD*, (fig. 4.)



5. A *Rhombus* is an equilateral rhomboid, having all its sides equal, but its angles oblique; as, *ABCD*, (fig. 5.)



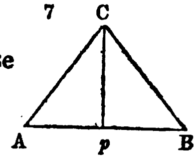
6. A *Trapezoid* is a quadrilateral figure, having only two of its sides parallel. The base of a figure is the side on which it stands, and the altitude is the perpendicular distance between its two parallel sides. Thus, *AB* is the base, and *ep* the height of the trapezoid, (fig. 6.) Any other four-sided figure is called a *Trapezium* or *Quadrilateral.*

II. When figures have more than four sides, they are classed under the head of *Polygons.* These again receive other particular names, according to the number of their sides or angles.

A regular Polygon has all its sides and angles equal. A Pentagon is a regular Polygon of five sides; a Hexagon has six sides; a Heptagon has seven sides; an Octagon has eight sides; a Nonagon has nine sides; a Decagon has ten sides; an Undecagon has eleven sides; and a Dodecagon has twelve sides.

III. A figure of three sides and angles is called a *Triangle,* and receives particular denominations from the relations of its sides and angles.

7



1. An equilateral triangle is that whose three sides are equal; as, *ABC*, (fig. 7.)

2. The *height* of a triangle is the length of a perpendicular drawn from one of the angles to the opposite side; as, *Cp*, (fig. 7.)

The height of a four-sided figure is the perpendicular distance between two of its parallel sides; as, *Dp*, (fig. 1.)

IV. The *area*, or *superficial* contents of any plane figure, is the measure of the space contained within the lines by which the figure is bounded.

In calculating the area or the contents of any plane figure, some particular portion of surface is fixed upon as the *measuring unit*, with which the figure is to be compared.

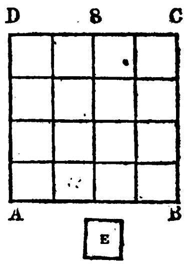
This is commonly a *square*, the side of which is the unit of length, being an *inch*, or a *foot*, or a *yard*, or any other fixed quantity, according to the measure peculiar to different artists.

The same holds true, also, when the figure is a square. So, the area of the rhombus or rhomboides is equal to that of a parallelogram of the same base and altitude.

Hence, the square foot, yard, &c., may be of any *shape* whatever, provided the foot contains 144 squares, each 1 inch square, and the yard 9 squares, each 1 foot square.

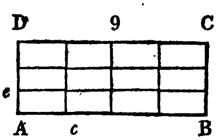
And hence, the area or quantity of surface contained in a figure, is said to be so many square inches, square feet, or square yards.

For the same reason, determining the quantity of surface in a figure is called *squaring it ;* that is, determining the square or number of squares to which it is equal.



Thus, if the figure to be measured be the square *ABCD*, in the adjoining figure, and the little square E, whose side is one inch, be the measuring unit fixed upon, then as often as the square E is contained in the whole figure, so many square inches it is said to contain, which, in the present case, we find to be 16. Again: supposing the figure to contain 4 square miles of land, then 4 miles square of land would be 4 times 4, or 16 square miles, making a difference of 12 square miles, (fig. 8.)

The fundamental problem in the mensuration of superficies, is the very simple one of determining the area of a *right parallelogram,* as has been shown above. The contents of other figures may readily be obtained by finding parallelograms which are equal to them.



If the parallelogram be divided into small parallelograms, by drawing lines as shown in the subjoined figure, it is obvious, on inspection, that the number of the little parallelograms must always be equal to the product of the length and breadth. If we count the parallelograms in the upper row of this figure, we find the number to be 4; in two rows, twice 4, or 8; in three rows, three times 4, or 12. Hence, to obtain the number of small parallelograms, *ce*, contained in the large parallelogram, *ABCD*, we have only to multiply the number of such small parallelograms contained in the side *AB*, into the number

of such contained in the side BC, which, in the present case, we find to be, $4\times3=12$.

ART. 3. The *superficial unit* is generally called by the same name as the *linear* unit, which forms the sides of the square. If the side be an inch, it is called a linear inch; the side of a square foot, a linear foot; the side of a square rod, a linear rod; and so of any other fixed quantity. It should be remarked, however, that there are some superficial measures which have no corresponding denominations of length with which to compare them. For instance, the *acre* is not a square which has a line of the same name for its side.

Hence, the utility of the following Tables, which contain the linear measure, in common use, with their corresponding square measures.

Linear Measure.		*Square Measure.*	
12 inches	= 1 foot,	144 inches	= 1 foot,
3 feet	= 1 yard,	9 feet	= 1 yard,
6 feet	= 1 fathom,	36 feet	= 1 fathom,
$16\frac{1}{2}$ feet	= 1 rod,	$272\frac{1}{4}$ feet	= 1 rod,
$5\frac{1}{2}$ yards	= 1 rod,	$30\frac{1}{4}$ yards	= 1 rod,
4 rods	= 1 chain,	16 rods	= 1 chain,
40 rods	= 1 furlong,	102400 rods	= 1 mile.
320 rods	= 1 mile.		

By reducing the denominations of square measure, it will readily be seen that

1 square mile=640 acres=102400 rods=27878400 ft.

1 acre=10 chains=160 rods=43560 feet.

From what has been already shown, we learn that multiplying the length of any square or parallelogram by its breadth, will give its square measure, or as it is sometimes

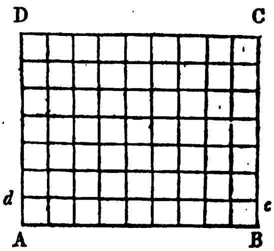
called, its square contents, as will be seen in the demonstration of the following problems.

PROBLEM I.

To find the area of a Four-sided Figure, whether it be a parallelogram, square, rhombus, or rhomboid.

Art. 4. *Rule.*—Multiply the length by the breadth or perpendicular height, and the product will be the area.

It is manifest that the number of square inches in the parallelogram *ABCD* is equal to the number of *linear* inches in the length *AB*, taken as many times as there are inches in the breadth *BC*. To obtain, then, the number of squares in the large parallelogram, we have only to multiply the number of squares in one of the small parallelograms (*ABcd*,) by the number of such parallelograms contained in the whole figure. Now, the number of square inches in one of the small parallelograms is 9, and the number of *linear* inches in the breadth *BC* is 7. Therefore the product of the length into the breadth is 63 square inches. It is therefore said concisely, that the area of a parallelogram is equal to the length multiplied into the breadth.



Example 1. How many square feet are there in a floor $23\frac{1}{2}$ feet long and 18 feet broad?

$$\begin{array}{lr} & 23.5 \\ \text{OPERATION.} & 18.0 \\ \hline & 188.00 \\ & 235 \\ \hline \textit{Ans.} & 423.00 \end{array}$$

Or thus, $23\frac{1}{2} \times 18 = 423$, *Ans.*

Ex. 2. What is the area of a parallelogram whose length is 12 feet 3 inches, and whose breadth is 8 feet 6 inches?

Ans. 104.125.

Ex. 3. How many square feet are there in the four sides of a room which is 22 feet long, 17 feet broad, and 11 feet high?

22 feet=length of one side.
11 feet=height of one side.

242=square feet in one side.
2

484=square feet in two sides.

Again, 17 feet=length of the short side.
And 11 feet=height of the short side.

187=square feet in the short side.
2

374= square feet in both short sides.

Then, 484+374=858 *sq. ft. Ans.*

NOTE.—The very convenient Arithmetical rule of Duodecimals should be thoroughly learned, (and it may be in a few hours of close application,) by every one who would make progress in Mathematical science.

Ex. 4. How many feet of glass are contained in a window 4 feet 11 inches high and 3 feet 5 inches broad?

Ans. 16 *ft.* 9′ 7″.

NOTE.—When the dimensions are given in feet and inches, the most convenient way of performing the operation is by the Arithmetical rule of Duodecimals, in which each inferior denomination is one-twelfth of the next higher.

Ex. 5. How many square yards of painting are there in a rhomboid, whose length is 37 feet, and height 5 feet 3 inches?

	FT. IN.	Or thus,
Thus,	37 0′	37 feet= 444 inches.
	5 3′	5 ft. 3 in.= 63 "
	185 0′	1332
	9 3′ 0″	2664
9 sq. ft.=1 sq. yd.	9)194 3′ 0″	1296)27972(21$\frac{7}{12}$
Ans. 21$\frac{7}{12}$.	21 7′ 0″	2592
		2052
144 sq. in.=1 sq. ft. and 9×144=1296.		1296
		$\frac{756}{1296}=\frac{7}{12}$.

Ex. 6. Herodotus estimated the largest and most remarkable of the Egyptian Pyramids to be 800 feet square at the base. Now, how long a road, 4 rods wide, would occupy as much land as the base of the pyramid?

Ans. 1 *mile*, 6 *fur.* 27$\frac{757}{1089}$ *rods.*

Art. 5. We have already seen that the area of any parallelogram is obtained by multiplying the length into the breadth. Now, if the area and one side of any parallelogram be given, the *other side* may be found by dividing the area by the given side. So, also, if the area of a square be given, either side may be found by *extracting the square root of the given area.* This is merely reversing the rule in Art. 4, where a given side is squared.

Ex. 1. What is the width of a street 12 rods long, which contains 40 square rods?

40÷12=3$\frac{1}{3}$ rods, or 55 feet, *Ans.*

Ex. 2. What is the side of a piece of land containing 1681 square rods? *Ans.* 41.

Ex. 3. How much carpeting 1 yard wide will cover a floor 18 feet long and 16$\frac{1}{2}$ feet wide?

18×16$\frac{1}{2}$=297 square feet.
9 sq. ft.=1 sq. yd. and 297÷9=33 yds.

Ex. 4. If a room be $16\frac{1}{2}$ feet wide and $18\frac{1}{2}$ feet long, how many yards of carpeting will cover the floor, allowing for a chimney 3 feet wide and $4\frac{1}{2}$ feet long ?

$$16\tfrac{1}{2}\times 18\tfrac{1}{2}=305\tfrac{1}{4}, \quad 305\tfrac{1}{4}\div 9=33\tfrac{11}{12}$$
$$3\times 4\tfrac{1}{2}=13\tfrac{1}{2} \quad 13\tfrac{1}{2}\div 9=\ \ 1\tfrac{6}{12}$$
$$32\tfrac{5}{12},\ \textit{Ans.}$$

Ex. 5. How many pieces of paper that are $\frac{1}{2}$ yard wide and 9 yards in a piece, will it require to paper a room 16 feet wide, 18 feet long, and 11 feet high, deducting for three doors, 7 feet 8 inches high and 4 feet 2 inches wide; also, two windows, 7 feet high and 4 feet 2 inches wide, and one fire-place, 6 feet long and 5 feet high ? *Ans.* $13\frac{8}{9}$.

Note.—First find the whole contents of the room as if there were no windows and doors; then find the contents of the windows and doors, and deduct the amount from the whole: the remainder will be the true answer in square feet. Next, divide the number of square feet by the number of square feet in a piece, and the quotient will be the number of pieces required.

Ex. 6. What is the side of a square which is equal to a parallelogram 936 feet long and 104 feet broad ?

Ans. 312.

Ex. 7. What is the side of a piece of land containing 169 square rods ? *Ans.* 13.

PROBLEM II.

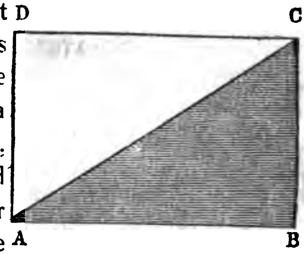
To find the area of a Triangle.

Art. 6. *Rule.*—Multiply the length of one of the sides by the perpendicular falling upon it, and half the product will be the area. Or multiply half the side by the perpendicular.

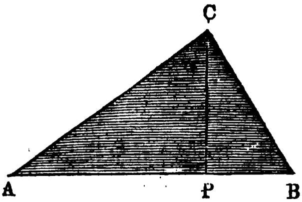
Note.—In a right angled triangle the longest side is called the *hypotenuse*, the next longest the *base*, and the shortest side the *perpendicular*.

The truth of this rule is evident, because any triangle is half of a parallelogram of the same base and altitude.

Thus, the area of the right angled triangle *ABC*, contains precisely half as much surface as would be contained in a square or parallelogram *ABCD*. two of whose sides are formed by the base and perpendicular of the triangle. Therefore, the area of a right angled triangle is found by multiplying together half the base *AB* and the perpendicular *BC*, or the side *AB* by half of *BC*.

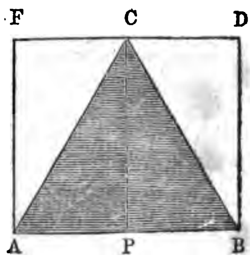


And whatever may be the form of a triangle, if it have not a right angle, it must be cut into two right angled triangles before it can be measured. This is done by letting fall a perpendicular from the opposite angle to the base, as *CP* in the subjoined figure, and then the area is found as before stated.



Ex. 1. What is the area of a triangle whose base is 20 feet, and altitude 10.25 feet? *Ans.* 102.5 *sq. ft.*

Once more: The area of the triangle *ABC* is precisely half of the parallelogram *ABDF*; for the surface contained in the right angled triangle *APC* is precisely equal to the surface contained in the right angled triangle *CFA*, and so *BPC*=*CDB*.



Ex. 2. What is the area of a triangular board, whose base is 4 feet 3 inches, and height 2 feet 11 inches ?

OPERATION.

	FT.	IN.	
	4	3′	
	2	11′	
	8	6′	
	3	10′	9″
Ans.	12	4′	9″

Ex. 3. What is the area of a triangle whose base is 18 feet 4 inches, and height 11 feet 10 inches ?

Ans. 108 *ft.* $5\frac{2}{3}$ *in.*

Ex. 4. How many square rods of land are there in a lot which is laid out in a right angled triangle, the base measuring 19 rods, and the perpendicular breadth 15 rods ?

Ans. 142.5.

Ex. 5. Find the number of square yards in a triangle whose base is 40 and altitude 30 feet. *Ans.* $66\frac{2}{3}$ *sq. yds.*

Ex. 6. What is the area of a triangle whose base is 72.7 yards, and altitude 36.5 yards ?

Ans. 1326.775 *sq. yds.*

ART. 7. If the three sides of a triangle are given, the area may be directly obtained by the following method :

To find the area of a Triangle from the length of its sides.

Rule.—I. Add together the lengths of the three sides, and take half their sum.

II. From this half sum subtract each side separately.

III. Multiply together the half sum and each of the three remainders, and extract the square root of the product; the quotient will be the required area of the triangle.

Ex. 1. If the sides of a triangle are 134.108 and 80 rods, what is the area ?

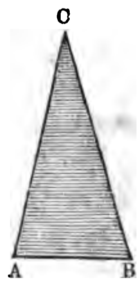
OPERATION.

134	161	161	161
108	134	108	80
80	27 1st rem.	53 2d rem.	81 3d rem.
2)322			
161 half sum.			

Then to obtain the products, we have $161\times27\times53\times81$ $=18661671$: from which we find area$=\sqrt{18661671},=4319$ square rods.

Ex. 2. What is the area of a triangle whose three sides are 52, 39 and 65 feet ? *Ans.* 1014 *sq. ft.*

An *Isosceles Triangle* is that which has only two sides equal; as *ABC* in the adjoining figure.



Ex. 3. What is the area of an isosceles triangle whose base is 20, and each of the equal sides 15? *Ans.* 111.803.

Ex. 4. How many square yards of plastering are there in a triangle whose sides are 30, 40, and 50 feet?

Ans. $66\frac{2}{3}$ *sq. yds.*

Ex. 5. Required the area of a triangular field, whose sides are 49, $50\frac{1}{4}$, and 25.69 chains?

Ans. 61 *A*, 1 *R.* 39.68 *P.*

Art. 8. ***To find the Hypotenuse of a right angled Triangle, when the base and perpendicular are known.***

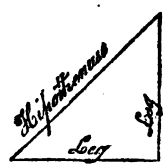
I. Square each of the sides separately.

II. Add together these squares.

III. Extract the square root of the sum, which will be the hypotenuse.

One of the properties of a right angled triangle is, that the square of either side is equivalent to the square of the hypotenuse diminished by the square of the other side; and the square of the hypotenuse is equal to the sum of the squares of the other two sides, usually called the legs of the triangle. This property is of great use, for by this means any two sides of a triangle being given, the other may be found by common Arithmetic.

Thus, in the right angled triangle *ABC*, the perpendicular *BC* and the base *AB* being given, the hypotenuse *AC* may be found by extracting the square root of the sum of the squares of the base and perpendicular.



Ex. 1. Let *AB* be 18 feet 8 inches, and *BC* 12 feet 6 inches.

FT. IN.	FT. IN.	
18 8	12 6	
18 8	12 6	
1504	756	Then, 353.44=sq. of base,
1504	252	158.76=sq. of perp.
188	126	512.20
353.44	158.76	

From which we find the hypotenuse $=\sqrt{512.20}=22.63+$

Note.—We first square each side, and then take the sum, of which we extract the square root, which gives *AB*=22.63.

Ex. 2. The wall of a building on the bank of a river is 120 feet high, and the breadth of the river 210 feet: what is the length of a line which will reach from the top of the wall to the opposite bank of the river?

Ans. 241.86 *ft.*

ART. 9. *To find one of the Legs when the Hypotenuse and the other Leg are known.*

Rule.—Subtract the square of the leg whose length is known, from the square of the hypotenuse, and the square root of their difference will be the answer.

Ex. 1. If AC (Prob. 2)=70 feet, and BC=60 feet, what will be the length of the side AB?

	OPERATION.
We first square the hypotenuse and the given side, and extract the square root of their difference.	$\overline{70}^2=4900$
	$\overline{60}^2=3600$
	Diff.$=1300$
	$AB=\sqrt{1300}=36.05.$

Ex. 2. The height of a precipice on the bank of a river is 103 feet, and a line of 320 feet in length will just reach from the top of it to the opposite bank; required the breadth of the river. *Ans.* 302.97 *ft.*

Ex. 3. The hypotenuse of a triangle is 53 yards, and the perpendicular 45 yards; what is the length of the base? *Ans.* 28 *yds.*

Ex. 4. A ladder 50 feet in length will reach to a window 30 feet from the ground on one side of the street, and by turning it over to the other side, it will reach a window 40 feet from the ground; required the breadth of the street.

Ans. 70 *ft.*

Art. 10. *If the area of an Equilateral Triangle be given, the sides may be obtained by the following*

Rule.—Divide the area by the decimal ,433013, and extract the square root of the quotient: the result will be the length of one side.

Ex. 1. The area of an equilateral triangle is one square chain: how many feet long is one side?

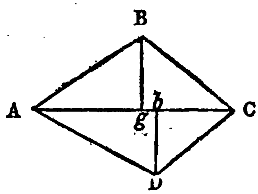
Ans. 100.29 *ft.*

PROBLEM III.

To find the area of a Trapezium.

Art. 11. *Rule.*—Divide the trapezium into triangles by drawing diagonals; and the sum of the areas of these triangles will be the area of the trapezium.

Ex. 1. In the irregular polygon *ABCD*, *AC*=24, and the perpendicular *Bg*=10, and *Db*=6; required the area.



Note.—A trapezium is an irregular figure of four unequal sides and angles.

$AC = 24$	24
$\frac{1}{2}$ of $Bg = 5$	$\frac{1}{2}$ of $Db = 3$
120	72

And the area$=120+72=192$

Ex. 2. Required the area of a trapezium whose diagonal is 78, and whose perpendiculars are 22 and 24.

Ans. 1794.

Ex. 3. What is the area of a trapezium whose diagonal is 42 feet, and the two perpendiculars 18 and 16 feet?

Ans. 714 *sq. ft.*

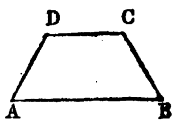
Ex. 4. What is the area of a trapezium in which the diagonal is 320,75 chains, and the two perpendiculars 69,73 chains and 130,27 chains? *Ans.* 3207 *A.* 2 *R.*

PROBLEM IV.

To find the area of a Trapezoid.

Art. 12. *Rule.*—Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product will be the area.

Ex. 1. Required the area of the trapezoid *ABCD*, having given *AB*=321.51 feet, *DC*=214.24 feet, and whose height is 171.16 feet.



We first find the sum of the sides, and then multiply it by the perpendicular height; after which, we divide the product by 2 for the area.

OPERATION.

321.51+214.24=535.75=the sum of the parallel sides.

Then,

535.75×171.16=91698.97,

And,

91698.97÷2=45849.485, *Ans.*

Ex. 2. How many square rods are contained in a field which has two parallel sides, 65 and 38 rods, and whose breadth is 27 rods? *Ans.* 139.05.

Ex. 3. Required the number of square feet in a trapezoid which has two parallel sides, 46 and 38 feet, distant from each other 27 feet. *Ans.* 1134.

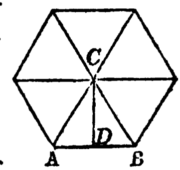
Ex. 4. What is the area of a trapezoid whose parallel sides are 10.50 chains and 18.25 chains, and whose perpendicular height is 16.80 chains?

Ans. 24 *A.* 0 *R.* 24 *P.*

PROBLEM V.

To find the area of a regular Polygon, or any regular figure.

ART. 13. *Rule* I.—Multiply one of its sides into half its perpendicular distance from the centre, and this product into the number of sides.



It is evident, on inspection, that a regular polygon contains as many equal triangles as the figure has sides.

Thus, the adjoining hexagon has six triangles, each equal to *ABC*. Now, the area of *ABC* is equal to the product of the side *AB* into $\frac{1}{2}$ of *CD*, (Art. 6.) The area of the whole, therefore, is equal to this product multiplied into the *number* of sides.

Ex. 1. Required the area of a regular hexagon, each of whose sides, *AB*, &c., is 45 feet, and the perpendicular, *CD*, 24 feet.

We first multiply one side by $\frac{1}{2}$ of the perpendicular *CD*, and that product by the number of sides: this gives the area.

OPERATION.

$45 \times 12 \times 6 = 3240$ *ft. Ans.*

Ex. 2. The side of a regular pentagon is 20 yards, and the perpendicular from the centre, on one of the sides, is 13.76382; required the area. *Ans.* 688.191 *sq. yds.*

Ex. 3. What is the area of a regular decagon whose sides are each 102 rods, and perpendicular from the centre 60 rods? *Ans.* 30600.

Ex. 4. Required the area of a regular decagon whose sides are each 87 feet, and perpendicular 28 feet.

Ans. 12180 *sq. ft.*

To facilitate the measurement of polygons, the following Table is constructed, showing the Multipliers of the ten regular polygons, when the sides of each is equal to 1: it also shows the length of the Radius of the inscribed circle.

Number of sides.	Names.	Area, or Multipliers.	Rad. of inscribed circle
3	Triangle,	0.4330127	0.2886751
4	Square,	1.	0.5000000
5	Pentagon,	1.7204774	0.6881910
6	Hexagon,	2.5980762	0.8660254
7	Heptagon,	3.6339124	1.0382617
8	Octagon,	4.8284271	1.2071068
9	Nonagon,	6.1818242	1.3737387
10	Decagon,	7.6942088	1.5388418
11	Undecagon,	9.3656404	1.2028437
12	Dodecagon,	11.1961524	1.8660254

Now, since the areas of similar polygons are to each other as the squares of their homologous sides, if the square of the side of a polygon be multiplied by the multiplier of the like figure, the product will be the area sought. And hence we have,

1^2 : tabular area, : : any side squared : area.

By this table may be calculated the area of any other polygon of the same number of sides with one of these. Hence, to find the area of any regular polygon, we have the following method, in addition to the rule already given:

Rule II.—1. Square the side of the polygon.

2. Multiply the square thus found by the tabular multi-

plier set opposite the polygon of the same number of sides, and the product will be the area.

Ex. 1. Required the area of a regular octagon whose side is 20 feet.

OPERATION.

$$\overline{20}^2=400$$

And the tabular multiplier or area $\Big\}$ $=4.8284271$

Hence, $4.8284271\times400=1931.3708400$, *Ans.*

Ex. 2. What is the area of a regular decagon whose side is 87 feet? *Ans.* 58237.46.

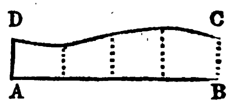
Ex. 3. The side of an undecagon is 40: what is its area? *Ans.* 14985.024.

Ex. 4. Required the area of a nonagon whose side is 50 feet. *Ans.* 15454.56.

Ex. 5. What is the area of a hexagon whose side is 25 feet? *Ans.* 1623.8.

PROBLEM VI.

To find the area of a long Irregular Figure, bounded on one side by a straight line.



ART. 14. *Rule.*—I. Measure the breadths in several places, and at equal distances from each other.

II. Add together all the different breadths, and half the sum of the two extremes.

III. Multiply this sum by the base line, and divide the product by the number of equal parts of the base.

Ex. 1. The breadths of an irregular figure, at five equidistant places, being 8.2, 7.4, 9.2, 10.2, 8.6, and the whole length 39, required the area.

OPERATION.

8.2	35.2=sum.
8.6	39
2)16.8=sum of extremes.	3168
8.4=mean of extremes.	1056
7.4	4)13728
9.2	3432 *Ans.*
10.2	
35.2 sum.	

Ex. 2. The length of an irregular figure being 84, and the breadths at six equidistant places, 17.4, 20.6, 14.2, 16.5 20.1, 24.4, what is the area? *Ans.* 1550.64.

The length of an irregular figure being 37.6, and the breadths, at nine equidistant places, 0.4.4, 6.5, 7.6, 5.4, 8, 5.2, 6.5 and 6.1, what is the area? *Ans.* 219.255.

Ex. 4. The length of an irregular field is 50 yards, and its breadths, at seven equidistant places, 5.5, 6.2, 7.3, 6, 7.5, 7, and 8.8 yards, what is its area?

Ans. 349,916 *sq. yds.*

NOTE.—If the perpendiculars or breadths be not at equal distances add them together, and divide their sum by the number of them, for the mean breadth; then multiply the mean breadth by the length, and the product will be the whole area not far from the truth.

PROMISCUOUS EXAMPLES.

1. What is the area of an equilateral triangle whose side is 20 rods? *Ans.* ——.

2. Required the number of square yards in an equilateral triangle, whose side is 10 yards. *Ans.* 43.3.

3. What is the area of a triangle whose base is 18 feet 4 inches, and height 11 feet 10 inches? *Ans.* 108.5$\frac{2}{3}$ *in.*

4. How many square yards are there in a trapezium, whose diagonal is 48 feet, and whose perpendiculars are 16 and 14 feet? *Ans.* ——

5. How many square rods in a trapezoid, whose parallel sides are 38 and 26 rods, and whose breadth or height is 18 rods? *Ans.* ——

6. Required the area of an octagon, whose side is 22 feet, and the perpendicular, from the centre on one of the sides, 12.478 feet. *Ans.* ——

7. What is the area of a pentagon, whose side is 8 feet 4 inches, and the perpendicular, from the centre on one of the sides, 4 feet? *Ans.* ——

8. If the length of an irregular figure be 43.5, and the breadths, at six equidistant places, be 4, 6.5, 6, 7.5, 8, and 8.5, what is the area? *Ans.* ——

9. There is a triangular lot of land whose base, or longest side, is $51\frac{1}{2}$ rods, and the perpendicular, from the opposite corner to the base, measures 44 rods; how many acres does it contain? *Ans.* ——

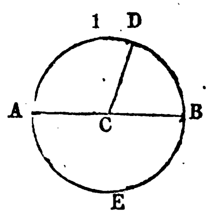
10. The length of an irregular piece of land being 21 chains, and the breadths, at six equidistant points, being 4.35, 5.15, 3.55, 4.12, 5.02, and 6.10 chains: required the area. *Ans.* ——

SECTION II.

MENSURATION OF THE CIRCLE AND ITS PARTS.

DEFINITIONS.

1



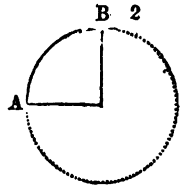
ART. 15.—1. A *Circle* is a plane figure bounded by a curve line, called the circumference, every part of which is equally distant from a certain point within, called the centre; *AEBD* (fig. 1) is a circle.

2. A *Diameter* of a circle is a straight line, passing through the centre and terminating at the circumference; as *AB* (fig. 1.)

3. A *Radius* or *Semi-Diameter* is a straight line, extending from the centre to the circumference; as *CA* or *CD* (fig. 1.)

4. A *Semi-circle* is one half of the circumference; as *ADB* (fig. 1.)

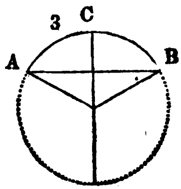
2



5. A *Quadrant* is one quarter of the circumference; as, *AB* (fig. 2.)

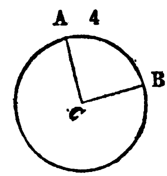
6. An *Arc* is any portion of the circumference; as *AC B* (fig. 3.)

7. A *Chord* is a straight line, which joins the two extremes of an arc; thus, *AB* is a chord of the arc *ACB*, (fig. 3.)

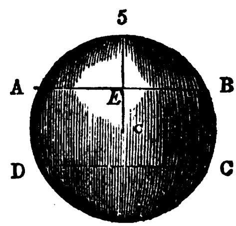


8. A *Circular Segment* is the space contained between an arc and its chord; as, *ACB* (fig. 3.) The chord is sometimes called the *base* of the segment. The *height* of the segment is the perpendicular from the middle of the base to the arc.

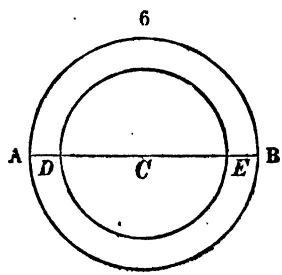
9. A *Circular Sector* is the space contained between an arc and the two radii, drawn from the extremes of the arc: thus, *CAB* (fig. 4) is a sector.



10. A *Circular Zone* is the space contained between two parallel chords which form its bases; thus, *ABCD* (fig. 5.) It is called the *middle* zone when the chords are of equal length.

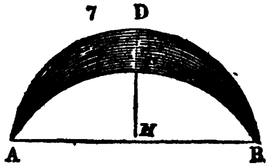


11. A *Circular Ring* is the space between the circumferences of two concentric circles; thus, *AB* and *DE* (fig. 6) is a circular ring, having a common centre, *C*.



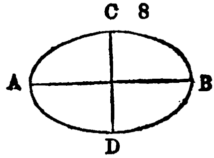
12. A *Lune* or *Crescent* is the space between two circular arcs, which intersect each other; thus, *ADB* and *ACB* (fig. 7) is a lune.

7



An *Ellipse* or *Oval* is a curve line, which returns into itself like a circle, but has two diameters of unequal length, the longest of which is called the transverse, and the shortest the conjugate axis; thus, *ABCD* (fig. 8) is an ellipse.

8



PROBLEM I.

To find the circumference of a Circle when the diameter is given.

Art. 16. *Rule* I.—Multiply the diameter by 3.14159, and the product will be the circumference. Or,

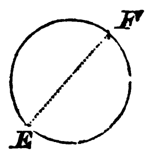
Rule II.—As 7 : 22 : : diameter to the circumference; that is, Multiply the diameter by 22 and divide the product by 7. Or,

Rule III.—Multiply the diameter by 355, and divide the product by 113.

Note.—The learner may have the curiosity to enquire why we use the number 3,14159. or 3,1416, as is sometimes used, instead of any other number. by which to multiply the diameter of a circle to find its area. It is the result of the efforts of the most distinguished mathematicians to Square the Circle. *Archimedes* made the earliest approximation to the ratio of the circumference of a circle to its diameter. He demonstrated that the ratio of the perimeter of a regular inscribed polygon of 96 sides, to the diameter of the circle. is greater than $3\frac{10}{71}$: 1; and that the ratio of the perimeter of a circumscribed polygon of 192 sides, to the diameter, is less than $3\frac{10}{71}$: 1; that is, than 22 : 7.

Metius next followed, and gave the ratio of 355 : 113, which is a little more accurate than any other expressed in small numbers.

Now, to show how they obtained these numbers, and how near they are to the truth, let us *suppose* the subjoined *Dodecagon* to contain 1556 sides instead of 12; we shall then have a regular polygon of 1556 sides *inscribed* in the circle, and if another similar polygon be *circumscribed* about the circle, it may be demonstrated that the *perimeter* of the inscribed polygon is 6,2831788, and that of the circumscribed polygon 6,2831928. Now, the circumference of the circle being *greater* than the perimeter of the inscribed polygon, and *less* than that of the circumscribed, it must consequently be greater than 6,2831788, and less than 6,2831928, and must therefore be nearly half their sum, which is 6.2831858. Hence, the circumference being 6,2831858, when the diameter is 2, it will be the half of that, or 3,1415928, when it is 1; to which the ratio, in the Rule, viz: 1 to 3,14159 very nearly.



Ex. 1. What is the circumference of a circle whose diameter EF is 24 feet?

In this operation we simply multiply the diameter, 24, by the number 3.14159, and the product gives the circumference.

OPERATION.

$24 \times 3.14159 = 75.39816$, which is the circumference.

Ex. 2. What is the circumference of the earth, the diameter being 7930 miles? *Ans.* 24912.8 *miles.*

Ex. 3. Required the circumference of a circle whose diameter is $73\frac{3}{4}$. *Ans.* 231.6924.

Ex. 4. What is the circumference of a circle whose diameter is 40 feet? *Ans.* 125.65.

PROBLEM II.

To find the Diameter of a Circle when the circumference is given.

ART. 17. *Rule* I.—Divide the circumference by 3.14159, and the quotient will be the diameter. Or,

Rule II.—Multiply the circumference by 7 and divide the product by 22. Or,

Rule III.—Multiply the circumference by 113 and divide the product by 355.

Ex. 1. If the circumference of a circle be 14 feet 5 inches, what is its diameter?

We simply divide the circumference by 3.14159, and the quotient, 4.21, is the diameter.	OPERATION. 14 ft. 5 in.=173 in. And 173÷3.14159=50.56 in. which=4.21 ft. the diameter.

Ex. 2. If the circumference of the Sun be 2800000 miles, what is his diameter? *Ans.* 891267 *miles.*

Ex. 3. What is the diameter of a cylinder whose circumference is 146.084? *Ans.* 46.5.

Ex. 4. What is the diameter of the Moon if her circumference be 6850 miles? *Ans.* 2180.

Ex. 5. Required the diameter of a tree whose circumference is 5½ feet. *Ans.* 21 *inches.*

ART. 18. The same result may be obtained more conveniently, by exchanging the *divisor*, 3.14159, for a *multiplier*, which will give the same answer.

Now, in the proportion 3.14159 : 1 : : Circ. : Diam. the fourth term, may be directly found by dividing the second by the first, and multiplying the quotient into the

third. Thus, 1÷3.14159=0.31831. Therefore, if the circumference of any circle be *multiplied* by the decimal .31831, the product will be the diameter.

In many cases there will be a decided saving of labor by exchanging the *divisor* for a *multiplier*, as will be seen in the following example.

Ex. 1. What is the diameter of a circle whose circumference is 50 ?

	OPERATION.
As multiplication is more easily performed than division, the first method is decidedly the more preferable.	50×.31831=15.91550, *Ans.* Or thus, 50÷3.14159=15.9155+

Ex. 2. The circumference of a circle is 69.115 yards: what is the diameter ? *Ans.* 22 *yds.*

Ex. 3. If the whole extent of the orbit of Saturn be 5650 million miles, how far is he from the Sun ?

Ans. 899225750 *miles.*

PROBLEM III.

To find the area of a Circle when the diameter and circumference are both known.

ART. 19. *Rule* I.—Multiply the square of the diameter by .7854. Or,

Rule II. Multiply the diameter into the circumference, and divide the product by 4; in either case the product will be the area.

The area of a circle is equal to the product of the diameter into the circumference.

Now, we have just seen, (Art. 18,) that if the diameter be 1, the circumference will be 3.14159, and one fourth of

this is 0.7854, nearly. Consequently, the area of any circle is found by multiplying the square of the diameter by .7854, which is the area of a circle whose diameter is 1.

Ex. 1. Required the area of a circle whose diameter is 623 feet.

We square the diameter, which gives us 388129, and this number we multiply into the decimal .7854, which gives the area.

OPERATION.

$$\begin{array}{r} \overline{623}^2 = 388129 \\ .7854 \\ \hline 1552516 \\ 1940645 \\ 3105032 \\ 2716903 \\ \hline 304836.5166 \end{array}$$ *Ans.*

Ex. 2. What is the area of a circular pond whose diameter is 40 rods?

$\overline{40}^2 = 1600$, and $1600 \times .7854 = 1256.64$; *Ans.*

Ex. 3. What is the area of a circle whose diameter is 33,25 inches? *Ans.* 868,30 *sq. in.*

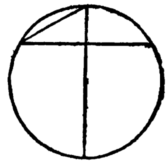
Ex. 4. How many square yards are there in a circle whose diameter is 5 feet? *Ans.* 2.18.

Ex. 5. What is the area of a circle whose diameter is 4.5? *Ans.* 15.904.

PROBLEM IV.

To find the area of a Circle when the Circumference only is given.

ART. 20. *Rule.*—Multiply the square of the circumference by the decimal .07958, and the product will be the area very nearly.



The truth of this will easily appear, by considering that if the circumference of a circle be 1, the diameter=0.31831, (Art. 18,) and $\frac{1}{4}$ of the product of this into the circumference is .07958, the area. But the areas of different circles being as the squares of their diameters, are also as the squares of their circumferences. Consequently, the area of a circle is readily found, by multiplying the square of the circumference by .07958.

Ex. 1. If the circumference of a circle be 136 feet what is the area?

We square the circumference, which gives us 18496, which, multiplied by the decimal. 07958, gives the area.

OPERATION.

$$
\begin{array}{r}
\overline{136}^2=18496 \\
.07958 \\
\hline
147968 \\
92480 \\
166464 \\
129472 \\
\hline
1471.91168
\end{array}
$$
Ans.

Ex. 2. What is the area of a circle whose circumference is 113? *Ans.* 1016.158.

Ex. 3. How many square feet are there in a circle whose circumference is 10.9956 yards? *Ans.* 86.5933.

Ex. 4. What is the area of a circle whose circumference is 67? *Ans.* 357.234.

Ex. 5. What is the area of a circle whose diameter is 39.34 inches? *Ans.* 1240.98 *sq. in.*

Ex. 7. Required the area of the two ends of a cylinder whose diameter is 3 feet. *Ans.* 7.068 *ft.*

PROBLEM V.

To find the area of a Circle when the Diameter only is known.

ART. 21. *Rule.*—Multiply the square of the diameter by the decimal .7854, and the product will be the area.

Ex. 1. What is the area of a circle whose diameter is 11 feet?

OPERATION.

$$\overline{11}^2=121\times.7854=95.0334,\ \textit{Ans.}$$

Ex. 2. What is the area of a circle whose diameter is 8.75 feet? *Ans.* 60.1320.

Ex. 3. Required the area of a circle whose diameter is 92.75 feet. *Ans.* 6756.436.

PROBLEM VI.

To find the diameter of a Circle when the Area only is known.

ART. 22. *Rule.*—Divide the area by the decimal .7854, and the square root of the quotient will be the diameter.

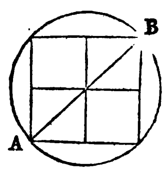
This is just the reverse of the rule in Art. 19.

Ex. 1. What is the diameter *AB* of a circle whose area is 380.1336?

OPERATION.

$$380.1336\div.7854=484$$

And $\sqrt{484}=22$



Ex. 2. If a horse be tied by the head with a cord fastened to a post, so as to be able to graze exactly two acres of meadow, how long must the cord be?

Ans. 10.0925 *rds.*

Ex. 3. There is a meadow of 10 acres in the form of a square, and a horse tied equidistant from each angle or corner; what must be the length of the rope that will permit the horse to graze over every part of the meadow?

Ans. 28.284+*rods.*

Ex. 4. If the area of my garden be 95.033 square rods, what is the circumference and diameter of a circular garden of equal contents?

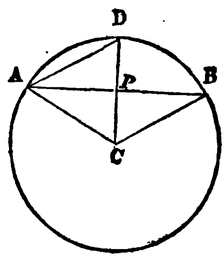
Ans. { 34.557. *circum. in rds.* / 11 *rds. diam.* }

PROBLEM VII.

To find the length of an Arc of a Circle, when the number of degrees which it contains and the Radius are known.

Art. 23. *Rule* I.—Multiply the number of degrees in the arc by the decimal .01745, and that product by the radius of the circle. Or,

Rule II.—As 3 is to the number of degrees in the arc, so is .05236 times the radius to its length.



Ex. 1. What is the length of an arc of 40 degrees, in a circle whose radius is 12 feet?

In this solution we simply multiply the given decimal by the number of degrees, and that product by the radius.

OPERATION.

$.01745 \times 40 \times 12 = 8.37600$,

= length of the arc.

Ex. 2. What is the length of an arc of 20 degrees, in a circle whose radius is 45 feet?

STATEMENT BY RULE II.

As 3 : 20 : : .05236×45 : 15.708, *Ans.*

Ex. 3. What is the length of an arc containing 15 degrees and 15 minutes, the diameter of the circle being 20 yards? *Ans.* 5.32225.

NOTE.—When the arc contains degrees and *minutes*, as in the last example, reduce the minutes to the decimal of a degree, which is done by dividing them by 60.

The length of an arc is frequently required when only the *chord* and the *height* are given, in which case the length of the arc may be found by the following approximating

Rule.—From 8 times the chord of half the arc, subtract the chord of the whole arc, and $\frac{1}{3}$ of the remainder will be the length of the arc, nearly.

Ex. 1. What is the length of an arc whose chord is 120, and whose height is 45?

NOTE.—The chord of half the arc is equal to the square root of the sum of the squares of the height and half the chord of the whole arc.

OPERATION.

$120 \div 2 = 60 = \frac{1}{2}$ chord of the arc.

$\overline{60}^2 = 3600$

$\overline{45}^2 = 2025$

$5625 = \left\{ \begin{array}{l}\text{sum of the squares}\\ \text{of base and perpen-}\\ \text{dicular of triangle.}\end{array}\right.$

And $\sqrt{5625} = 75 = \left\{ \begin{array}{l}\text{the chord of}\\ \text{half the arc.}\end{array}\right.$

Then,

$75 \times 8 = 600$

120 = chord of whole arc.

$480 \div 3 = 160$, *Ans.*

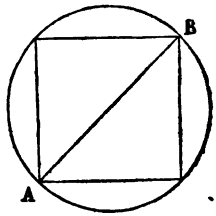
PROBLEM VIII.

To find the side of a Square inscribed in a Circle, from its circumference or Diameter.

Art. 24. *Rule.*—I. Multiply the diameter by .7071 =the side of the inscribed square.

II. Multiply the circumference by .2251=side of the inscribed square.

Note.—The *area* of a circle is to the area of the *circumscribed square* as .7854 is to 1, and to that of the *inscribed square* as .7854 is to ½. Consequently, the square within the circle is precisely half of the square without.



Ex. 1. What is the side of a square inscribed in a circle whose diameter *AB* is 200 feet?

OPERATION.

$.7071 \times 200 = 141.4200$=the side of the inscribed square.

Ex. 2. What is the *area* of a square inscribed in a circle whose area is 159?

$$.7854 : \tfrac{1}{2} :: 159 : 101.22 = \text{area.}$$

Ex. 3. What is the area of a square circumscribed about a circle whose area is 159?

$$.7854 : 1 :: 159 : 202.44.$$

Ex. 4. The circumference of a circular walk is 780: what is the side of an inscribed square? *Ans.* 175.578.

Ex. 5. The circumference of a circular pond is 312 feet: what is the side of the largest square sheet of ice which can be cut from it when frozen over? *Ans.* 70.2312 *ft.*

Ex. 6. The circumference of a circle is 715: what is the side of an inscribed square? *Ans.* 100.9445.

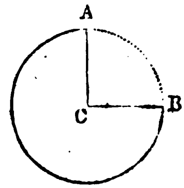
PROBLEM IX.

To find the area of the Sector of a Circle.

Art. 25. *Rule.*—I. Find the length of the arc by Art. 23.

II. Multiply the length of the arc thus found, by half the length of the radius, and the product will be the area.

Or, As 360 degrees is to the number of degrees in the arc of the sector, so is the area of the circle to the area of the sector.



Ex. 1. If the *arc AB* be 120 degrees, and the diameter of the circle 226 degrees, what is the area of the sector?

Remark, 113 is $\frac{1}{2}$ the diameter, (Art. 23,) and $56\frac{1}{2}$ is $\frac{1}{2}$ the radius according to the above rule.

OPERATION.

First, $.01745 \times 120 \times 113 = 236.622$

Then, $236.622 \times 56\frac{1}{2} = 13369.142$

Note.—It is manifest that the area of the sector has the same ratio to the area of the circle which the number of *degrees* in the arc has to the number of degrees in the whole circumference.

Ex. 2. What is the area of a sector of a circle, in which the radius is 25 and the arc of 26 degrees? (See Art. 23.)
Ans. 141.8

Ex. 3. Required the area of a semi-circle, in which the radius is 13. *Ans.* 265.4143.

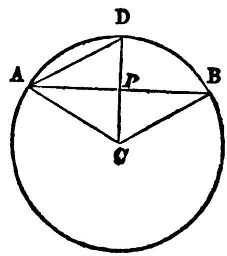
Ex. 4. What is the area of a circular sector, when the length of the arc is 650 feet, and the radius 325?
Ans. 105625 *sq. ft.*

PROBLEM X.

To find the area of the Segment of a Circle.

ART. 26. *Rule.*—I. To the chord of the whole arc add $\frac{1}{3}$ of the chord of half the arc.

II. Then multiply the sum by the versed sine, or height of the segment, and $\frac{4}{10}$ of the product will be the area of the segment, very nearly.



EX. 1. Required the area of a circular segment whose chord $AB=24$ feet, and whose radius $CA=20$ feet?

OPERATION.

First, $\overline{CA}^2-\overline{AP}^2=\overline{CP}^2=\sqrt{400-144}=16=CP$

Then, $CD-CP=DP=20-16=4$=height of segment.

Now, $\overline{AP}^2+\overline{PD}^2=\overline{AD}^2=\sqrt{144+16}=12.64911$=chord AD

And, 24. =the chord of the segment.

12.64911=chord of $\frac{1}{2}$ the segment.

4.21637=$\frac{1}{3}$ of the chord of $\frac{1}{2}$ the arc.

40.86548

4=the height of the segment.

163.46192

4

10)653.84768

65.384768=area of the segment.

Ex. 2. Required the area of a circular segment whose height is 19.2 and base or chord 70. *Ans.* 947.86.

$$\sqrt{35+19.2} = AD.$$

Ex. 3. What is the area of a circular segment whose chord is 16, and the diameter of the circle 20?

Ans. 44.680.

Ex. 4. If the base or chord of a segment be 10 feet, and the radius of the circle 12 feet, what is the area of the segment? *Ans.* 7.35 *sq. ft.*

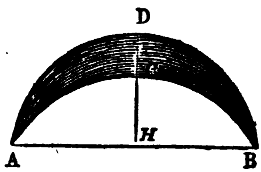
NOTE.—The following rule is given in Day's Mathematics: Find the area of the sector which has the same arc, and also the area of the triangle formed by the chord of the segment and the radii of the sector. Then, if the segment be less than a semi-circle, subtract the area of the triangle from the area of the sector. But if the segment be greater than a semi-circle, add the area of the triangle to the area of the sector.

PROBLEM XI.

To find the area of a Lune, or Crescent.

ART. 27. *Rule.*—Find the difference of the two segments which are between the arcs of the crescent and its chord, for the area.

Ex. 1. The chord of two segments AB is 72, and the height of the greater segment HD is 30, and of the less HC 20, what is the area of the crescent? *Ans.* 614.8692.



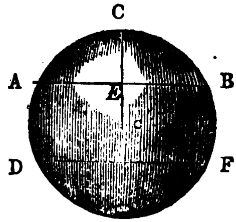
Ex. 2. If the chord AB be 88, the height HC 20, and the height HD 40; what is the area of the crescent, ADB and ACB? *Ans.* 1478.

NOTE.—For the demonstration of the above examples consult Art. 26.

PROBLEM XII.

To find the area of a Circular Zone.

Art. 28. *Rule.*—From the area of the whole circle subtract the areas of the two segments on the sides of the zone.



If from the whole circle there be taken the two segments *ABC* and *DFG*, there will remain the circular zone *ABFD*.

Ex. 1. What is the area of the zone *ABFD*, if *AB* is 7.75, *DF*, 6.93, and the diameter of the circle 8 ?

OPERATION.

50.26=area of the whole circle.
17.23=area of the segment *ABC*.
9.82=area of the segment *DFG*.
27.05

And, 50.26—27.05=23.21=area of the zone *ABFD*.

PROBLEM XIII.

To find the area of a Ring included between the circumferences of two Concentric Circles.

Art. 29. *Rule.*—I. Square the diameter of each circle, and subtract the square of the less from that of the greater.

II. Multiply the difference of the squares by the decimal .7854, and the product will be the area. Or,

III. Multiply the product of the *sum* and *difference* of the two diameters by .7854.

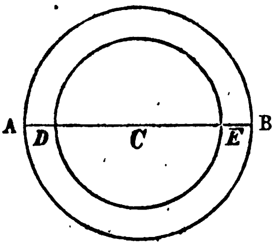
Ex. 1. If the diameter of the outer circle *AB* be 221, and the inner circle *DE*, 106, what is the area of the ring ?

OPERATION.

First, $\overline{221}^2 \times .7854 = 38359.72$

And, $\overline{106}^2 \times .7854 = 8824.75$

Ans. 29524.97



That is, the area of each of these circles is equal to the square of the diameter multiplied by .7854 (Art. 19.) And the difference of these squares is equal to the product of the *sum* and *difference* of the diameters. Therefore, the area of the ring is equal to the product of the sum and difference of the two diameters, multipled by .7854.

Ex. 2. The diameter of the inner circle is 12 rods, and the outer, 20 rods; required the area of the ring.

Ans. 201.06 *rods.*

Ex. 3. Supposing the diameter of Saturn's larger ring to be 205.000 and the smaller one 190.000 miles, required the number of square miles on one side of the ring.

Ans. 4.653.495.000.

Ex. 4. Two diameters are 21.75 and 9.5; what is the area of the circular ring? *Ans.* 300.66.

Ex. 5. If two diameters are $4\frac{1}{8}$ and $7\frac{3}{4}$, what is the area of the ring? *Ans.* 33.80.

PROBLEM XIV

To find the area of an Ellipse.

Art. 30. *Rule.*—Multiply the longer axis by the shorter, and the product, multiplied by the decimal .7854, will be the area required.

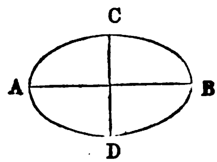
A common and more scientific name for the longer axis of an ellipse, is the *transverse, or major*, and for the shorter, the *conjugate* or *minor*.

Ex. 1 What is the area of an ellipse whose longer axis AB is 70 feet, and whose shorter DC is 50 feet ?

OPERATION.

$AB \times DE = 70 \times 50 = 3500$

Then, $3500 \times ,7854 = 2748.9 =$ area.



Ex. 2. What is the area of an ellipse whose axes are 16 and 12 ? *Ans.* 150.79.

PROBLEM XV.

To find the Circumference of an Ellipse.

Art. 31. *Rule.*—Square the two axes, and multiply the square root of half their sum by 3.14159 ; the product will be the circumference, nearly.

Ex. 1. What is the circumference of an ellipse whose transverse and conjugate axes are 16 and 18 feet ?

OPERATION.

$\overline{16}^2 + \overline{18}^2 = 580 =$ sum of the squares of the axes.

And, $290 =$ half sum.

Then, $290 \times 3.14159 = 911.064 =$ circumference.

Ex. 2. Required the circumference of an ellipse whose axes are 24 and 20 feet. *Ans.* 69.39.

PROBLEM XVI.

To find the area of an Elliptic Segment cut off by a line perpendicular to either axis.

Art. 32. *Rule.*—Find the area of a corresponding cir-

cular segment having the same height and the same vertical axis or diameter. Then say, as the vertical axis is to the other axis, parallel to the segment's base, so is the area of the circular segment before found, to the area of the elliptic segment sought.

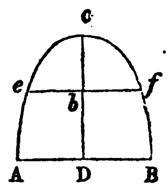
Ex. 1. What is the area of an elliptic segment, cut off parallel to the shorter axis, whose height is 10 and whose axes are 25 and 35? *Ans.* 162.03.

Ex. 2. What is the area of an elliptic segment, cut off parallel to the longer axis; whose height is 5 and the axes 25 and 35? *Ans.* 97.84.

PROBLEM XVII.

To find the area of a Parabola.

Art. 33. *Rule.*—Multiply the base by the height, and two-thirds of the product will be the area.



Ex. 1. What is the area of a parabola whose base *AB* is 26 inches, and height *Dc* 9 feet?

OPERATION.

We simply multiply the base and height together, and divide by $\frac{2}{3}$ for the answer.

First, 9 ft.=108 inches=height.
26 " =base.

648
216

$\frac{2}{3}$)2808 } =product of base and height.
2

3)5616

1872=area in sq. inches.

And, 1872÷144=13 *ft. Ans.*

Ex. 2. What is the area of a parabola whose base is 12 feet and height 18 feet? *Ans.* 144 *ft.*

Ex. 3. What is the area of a parabola whose base is $4\frac{1}{2}$ feet and height $10\frac{1}{2}$ feet? *Ans.* 31.50 *ft.*

PROBLEM XVIII.

To find the area of a Frustrum of a Parabola, cut off by a line drawn parallel to the base.

ART. 34. *Rule.*—Multiply the difference of the cubes of the two ends of the frustrum by twice its altitude, and divide the product by three times the difference of their squares.

Ex. 1. What is the area of a frustrum of a parabola whose height *Db* is 12 feet, and its upper end *ef* 12 feet, and base *AB* 20 feet? (See last problem.)

OPERATION.

$\overline{20}^2=400$ $\qquad$ $\overline{20}^3=8000$

$\overline{12}^2=144$ $\qquad$ $\overline{12}^3=1728$

256=diff. of their squares $\qquad$ 6272

3 $\qquad$ 24=twice the height.

768 $\qquad$ 25088

12544

768)150528(196 *ft. Ans.*

768

7372

6912

4608

4608

Ex. 2. What is the area of a frustrum of a parabola whose height is 8 feet, and its ends 16 and 10 feet?

Ans. 105.8.

PROBLEM XIX.

To find the area of a Hyperbola.

Art. 35. *Rule.*—1. To five-sevenths of the abscissa add the transverse diameter; multiply the sum by the abscissa, and extract the square root of the product.

II. Then, multiply the transverse diameter by the abscissa, and extract the square root of that product.

III. Then, to 21 times the first root, add 4 times the second root; multiply the sum by double the product of the conjugate and abscissa, and divide by 75 times the transverse; this will give the area, nearly.

Ex. 1. If the base of a hyperbola be 24 feet, the height 10, and the transverse axis 30; what is the area?

Ans. 151.69.

PROMISCUOUS EXAMPLES.

1. What is the area of a circle whose diameter is $26\frac{3}{8}$? *Ans.* 546.35.

2. What is the circumference of a circle whose diameter is $14\frac{5}{8}$? *Ans.* 45.94.

3. Required the area and circumference of a circle whose diameter is $56\frac{1}{8}$. *Ans.* 2474.0=*area.*
176.3=*circum.*

4. If the area of a circle be 2839.2, what is its diameter? *Ans.* 60.125.

5. What is the length of an arc of 30 degrees in a circle whose radius is 58 feet? *Ans.* 30.36.

6. What is the length of the side of a square, inscribed in a circle whose circumference is 300 feet? *Ans.* ——

7. If the diameters of two circles are 16 and 10, what will be the area included between the circumferences?

Ans. 122.52.

8. What will be the expense of papering the sides of a room, at 10 cents a square yard, if the room be 21 feet long, 18 feet broad, and 12 feet high, and if there be deducted 3 windows, each 5 feet by 3, two doors, 8 feet by $4\frac{1}{2}$, and one fire place, 6 feet by $4\frac{1}{2}$? *Ans.* $8.80.

9. What is the area of a circular segment whose height is 9 and base 24? *Ans.* ——

10. If a circular piece of land be enclosed by a fence, in which 10 rails make a rod in length, and if the field contains as many square rods as there are rails in the fence, what is the value of the land at 120 dollars per acre?

Ans. $942.48.

11. What is the area of a square inscribed in a circle whose diameter is 36 feet? *Ans.* ——

SECTION III.

MENSURATION OF SOLIDS BOUNDED BY PLANE SURFACES.

DEFINITIONS.

ART. 36. Mensuration of Solids is divided into two parts:

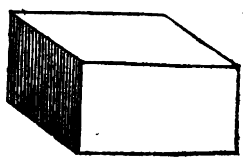
I. The mensuration of the surfaces of solids;

II. The mensuration of their solidities. (Art. 1.)

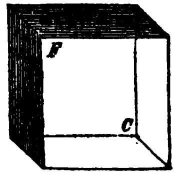
It has already been shown (Art. 2, Def. IV.) that the unit of measure for plane surfaces is a *square*, whose side is a *foot*, a *yard*, or any other fixed quantity.

1. A *Prism* is a solid whose ends are parallel, similar and equal, and the sides, connecting these are parallelograms. A prism takes particular names according to the figure of its base, whether triangular, square, rectangular, pentagonal, &c. The parallel planes are sometimes called *bases* or *ends*. The perpendicular distance between the bases is called the *height* of the prism.

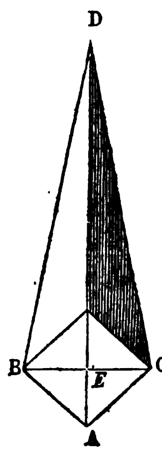
2. A *Parallelopiped* is a prism bounded by six quadrilateral planes, every opposite two of which are equal and parallel.



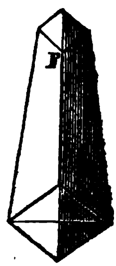
3. A *Cube* is a right prism, bounded by six equal square faces, of which any two, opposite to each other, are parallel.



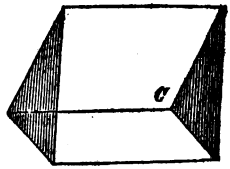
4. A *Pyramid* is a solid whose base is any plane figure, and whose sides are triangles, having all their vertices meeting together in a point above the base, called the *vertex* of the pyramid. The perpendicular distance from the vertex to the plane of the base is the *height* of the pyramid; as, *DE*. The *slant height* of a pyramid is a line drawn from the vertex to the middle of one of the sides of the base. A pyramid, like the prism, takes particular names from the figure of the base, according as it is square, triangular or polygonal.



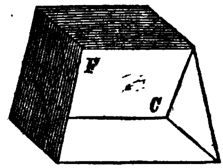
5. A *Frustrum* or *Trunk* of a pyramid is a portion of the solid that remains after any part has been cut off parallel to the base. The *height* of the frustrum is a line drawn through the centre of the pyramid from the centres of the two parallel planes. The *slant height* is a line passing on the surface of the frustrum through the middle of either of its sides.



6. A *Wedge* is a solid of five sides, two of which are rhomboidal, and meet in an edge, a rectangular base, and two triangular ends. The *height* of the wedge is the perpendicular distance between the edge and the plane of the base.



7. A *Prismoid* is a solid whose ends or bases are parallel, but not similar, and whose sides are quadrilateral.



8. *Surface* is the exterior part of any thing that has length and breadth, the limits that terminate a solid. *Convex* and *lateral* surface, are sometimes used synonymously in Mathematics.

PROBLEM I.

To find the Lateral Surface of a Right Prism.

ART. 37. *Rule.*—Multiply the perimeter of the base into the altitude, and the product will be the convex surface. When the *entire* surface of the prism is required, add to the convex surface the area of the bases. Hence, the superfices of any solid, bounded by planes, is equal to the sum of the areas of all its sides.

It is manifest that each of the sides of the prism is a parallelogram, whose area is equal to the product of the length into the breadth. Now, since the breadth is only one side of the base, therefore the sum of all the breadths is equal to the perimeter of the base.

Ex. 1. What is the entire surface of a regular prism, whose base is a regular pentagon, each side of which is 20 feet, and whose altitude is 50 feet ?

OPERATION.

$20\times5=100$=perimeter.
50=height.

5000 sq. ft. which is the convex surface.

We have for the area of the end (by Art. 13)
$\overline{20}^2\times$tabular number, or $400\times1.720477=688.1908$.

Then, 688.1908=area of one end.
688.1908= " "

1376.3816=area of two ends.
5000 =area of convex surface.

6376.3816=entire surface.

Ex. 2. Required the lateral surface of a prism whose base is a regular hexagon, and whose sides are each 2 feet 3 inches, the height being 11 feet ? *Ans.* 216 *sq. ft.*

Ex. 3. What is the entire surface of a triangular prism, whose base is an equilateral triangle, having each of its sides equal to 18 inches, and altitude 20 feet ?

Ans. 91.949 *sq. ft.*

Ex. 4. What is the surface of a regular Heptagonal prism, each side of whose base is 16 and altitude 15 feet ?

Ans. 1680 *sq. ft.*

PROBLEM II.

To find the Solidity of a Prism.

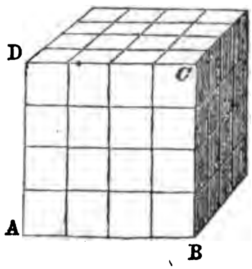
ART. 38. *Rule.*—Multiply the area of the base by the perpendicular height, and the product will be the solid contents.

NOTE.—The above rule holds true whatever the figure of the base may be, whether right or oblique parallelopipedons, cubes, &c.

As surfaces are measured by comparing them with a right parallelogram, so solids are measured by comparing them with a right parallelopipedon.

If the base of a right parallelopipedon be given, it is manifest that the number of cubic feet contained in one foot of the height, is equal to the number of square feet in the base. And if the solid be of any other height whatever, instead of one foot, the contents will be in the same ratio.

Let us illustrate this by an example: Let *ABCD* be the base of a right parallelopipedon, and suppose *AB* & *BC* each $=4$ feet. Then, the number of square feet in the base *ABCD* will be equal to $4\times4=16$ square feet.



Now, it is manifest that a parallelopipedon 1 foot in height contains 16 cubic feet; and were it 2 feet in height, it would contain 32 cubic feet, &c.

That is, the contents of any parallelopiped is found by multiplying the area of the base by the altitude of that solid. So, on the other hand, if the solid contents of a *cubic body* be given, the length of the edges may be found by *extracting the cube root* of the given solid.

Ex. 1. What is the solidity of a wall 28 feet long, 12 feet high, and 3 feet 4 inches thick?

We merely multiply the length, height, and thickness together, to find the solidity.

OPERATION.

FT.	
28	=length.
12	=height.
336	
$3\frac{1}{3}$	=thickness.
908	
112	
1020	=solidity.

Ex. 2. What is the solidity of a regular pentagonal prism, whose altitude is 20, and each side of the base 15 feet? *Ans.* 7742.1443.

Ex. 3. Required the capacity of a cubical vessel which is 3 feet 5 inches deep.

OPERATION BY DUODECIMALS.

$$\begin{array}{rllll} 3 & 5' & & \\ 3 & 5' & & \\ \hline 9 & 3' & & \\ 1 & 5' & 1'' & \\ \hline 10 & 8' & 1'' & \\ 3 & 5' & & \\ \hline 32 & 0' & 3'' & \\ 4 & 5' & 4'' & 5''' \\ \hline 36f & 5' & 7'' & 5''' \end{array}$$

Ex. 4. A cellar was dug, whose length was 49 feet 6 inches, breadth 23 feet 9 inches, and depth 9 feet 5 inches; how many solid yards of earth were taken out of it?

Ans. 410 *yards.*

Ex. 5. Required the solidity of a triangular prism whose altitude is 10 feet, and the three sides of its triangular base 3, 4, and 5 feet. *Ans.* 60 *solid ft.*

Art. 39. The capacity of a vessel in gallons or bushels of any given dimensions, may be readily ascertained by calculating its contents in *inches*, and then dividing the contents by the number of cubic inches in one gallon or bushel.

Ex. 1. Required the number of ale gallons there are in a *cistern* which is 9 feet 8 inches deep, and whose base is 5 feet 4 inches square? *Ans.* 16848 *gall.*

In this operation we simply calculate the capacity of the cistern in cubic inches, and then divide by the number of cubic inches contained in 1 gallon.

OPERATION.

First, 5 ft. 4 in.=64 inches.

And, $\overline{64}^2=$ 4096

9 ft. 8 in.= 116

Then, $4096\times116=475136$

And, $475136\div282$ (cubic inches in gall. ale)=1684.8, *Ans.*

Ex. 2. How many ale gallons are there in a cistern whose base is 5 feet 3 inches square, and which is 9 feet 8 inches deep? *Ans.* 1632.6.

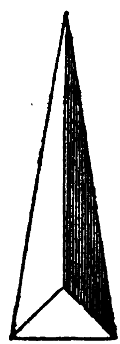
Ex. 3. How many *wine* gallons are there in a cistern 11 feet 9 inches long, 4 feet 9 inches wide, and 3 feet deep? *Ans.* 1252.5.

PROBLEM III.

To find the Lateral Surface of a regular Pyramid.

Art. 40. *Rule.*—Multiply the perimeter of the base by the slant height, and half the product will be the surface. If the whole surface be required, add to this the area of the base.

Ex. 1. What is the lateral surface of a regular triangular pyramid whose slant height is 20 feet, and the sides of whose base are each 8 feet?



OPERATION.

$8\times3=24$=perimeter of the base.

20=slant height.

2)480

240=lateral surface.

Ex. 2. What is the entire surface of a regular pyramid whose slant height is 15 feet, and whose base is a regular pentagon, each side of which is 25 feet?

Ans. 2012.798 *sq. ft.*

Ex. 3. Required the convex surface, and also the entire surface, of a regular pentagonal pyramid, whose slant height is 45, and the sides of whose base are each 15 feet.

Ans. 1687.5=*convex surface.*
387.107=*area of the base.*
2074.607 *sq. ft.=whole surface.*

PROBLEM IV.

To find the Lateral Surface of the Frustrum of a regular Pyramid.

Art. 41. *Rule.*—Multiply the perimeters of the two ends by the slant height of the frustrum, and half the product will be the surface required. To this add the surface of the two ends when the entire surface is required.

Ex. 1. What is the lateral surface of the frustrum of a regular octagonal pyramid whose slant height is 42 feet, and the sides of the lower base 5 feet each, and of the upper base, 3 feet each.?

OPERATION.

First, 5×8=40=perimeter of lower base.
3×8=24= " upper "
64=sum of the two ends.
42=slant height.
128
256
2)2688
1344=area of lateral surface.

Ex. 2. How many square feet are there in the lateral surface of the frustrum of a square pyramid, whose slant height is 10 feet, each side of the lower base 3 feet 4 inches, and each side of the upper base 2 feet 2 inches?

Ans. 110.

Ex. 3. If the slant height of the frustrum of a hexagonal pyramid be 48, each side of the lower base 26, and each side of the upper base 16 feet, what is the lateral surface?

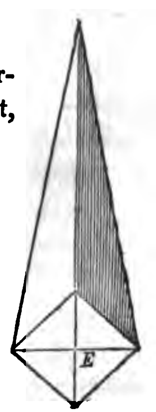
Ans. 6048 *sq. ft.*

PROBLEM V.

To find the Solidity of a Pyramid.

Art. 42. *Rule.*—Find the area of the base, and multiply that area by $\frac{1}{3}$ of the height.

This rule follows from that of the *prism*, because any pyramid is $\frac{1}{3}$ of a prism of the same base and altitude. It is manifest, therefore, that the solidity of a pyramid, whether right or oblique, is equal to the product of the area of the base into $\frac{1}{3}$ of the perpendicular height.



Ex. 1. What is the solidity of a square pyramid the sides of whose base are each 30 feet, and its perpendicular height 25 feet?

OPERATION.

$$\begin{array}{ll} \text{First, } 30\times30=900=\text{area of the base.} & \\ 25\div3= \quad 8\frac{1}{3} & \\ \hline \qquad 7200 & \\ \qquad\ \ 300 & \\ \hline \qquad 7500=\text{solidity.} & \end{array}$$

Ex. 2. How many solid feet in a triangular pyramid the altitude of which is 14 feet 6 inches, and the three sides of its base, 5, 6 and 7 feet? *Ans.* 71.0352.

Ex. 3. What are the solid contents of a triangular pyramid, the sides of whose base are each 3 feet, and height 30 feet? (Find the area of the base by Art. 7.)

Ans. 38.97.

Note.—A *Triangular Pyramid*, whose height, and each side of its triangular base, are each 12 inches, is *nearly* $\frac{1}{7}$ of the cube whose linear edge is equal to a side of the triangular base, and contains 249.413 cubic inches. This is a short method of obtaining the solidity of the pyramid, and is sufficiently accurate for all practical purposes, as may be seen by applying it to the solution of the above proposition.

Thus, $3\times3\times30\div7=38.571$.

Ex. 4. What is the solidity of a regular pentagonal pyramid, its altitude being 12 feet, and each side of its base 2 feet? *Ans. 27.52 solid ft.*

PROBLEM VI.

To find the Solidity of the Frustrum of a Pyramid.

Art. 43. *Rule.*—To the areas of the two ends of the frustrum, add the square root of their product, and this sum, multiplied by $\frac{1}{3}$ of the perpendicular height, will give the solid contents.

Note.—This Rule holds equally true to a pyramid of any form. For the solidities of pyramids are equal when they have equal heights and bases, whatever be the figure of their bases.

Ex. 1. In the frustrum of a pyramid, (Art. 40,) one end of which is 9 feet square, the other end 6 feet square, and the height 40 feet, what is the solidity?

OPERATION.

First, $9\times9=81$=area of base.

$6\times6=36$=area of upper base.

$\overline{117}$

$81\times36=2916$, and $\sqrt{2916}=54$=sq. root of prod. of 2 areas.

Then, $117+54=171$

$13\frac{1}{3}=\frac{1}{3}$ of the height.

$$\begin{array}{r} 513 \\ 171 \\ 57 \\ \hline 2280 \end{array}$$
=solidity.

Ex. 2. What is the solidity of the frustrum of a regular pentagonal pyramid, whose altitude is 5 feet, each side of one end 18 inches, and each side of the other end 6 inches?

Ans. 9.31925 *cubic ft.*

Ex. 3. If the height of a frustrum of a pyramid be 24, and the areas of the two ends 441 and 121, what is the solidity?

Ans. 6344.

Ex. 4. What is the solidity of a frustrum of a hexagonal pyramid, whose height is 48, each side of the lower base 26, and each side of the upper base 16? *Ans.* 56034.

Ex. 5. What is the solidity of a squared piece of timber, whose length is 18 feet, each side of the lower end 18 inches, and each side of the smaller, 12 inches?

Ans. 28.5 *cubic ft.*

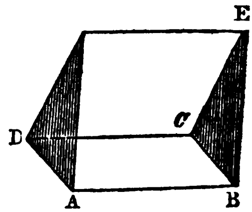
PROBLEM VII.

To find the Solidity of a Wedge.

ART. 44. *Rule.*—I. To the length of the edge of the wedge add twice the length of the base.

II. Then multiply this sum by the height of the wedge, and the breadth of the base; and $\frac{1}{6}$ of the product will be the solid contents.

Ex. 1. Required the solidity of a wedge whose base *AB* is 27 feet, *BC*, 8 feet, and whose edge *CE* is 36 feet, and the perpendicular height 42 feet.



OPERATION.

First, 36=length of the base.
54=twice the length of the base.

90
42=height of the wedge.

180
360

And, $3780 \times 8 \div 6 = 5040$=solidity of wedge.

Ex. 2. How many solid feet are there in a wedge whose base is 5 feet 3 inches long and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 3 inches? *Ans.* 3.9375 *cubic ft.*

Ex. 3. What is the solidity of a wedge, the length and breadth of whose base are 35 and 15 inches, and the length of the edge is 55 inches, and whose perpendicular height is 18 inches? *Ans.* 32552 *cubic ft.*

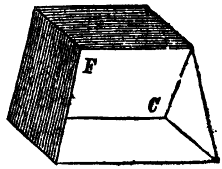
PROBLEM VIII.

To find the Solidity of a Rectangular Prismoid.

ART. 45. *Rule.*—To the sum of the areas of the two ends add four times the area of a section parallel to and

equally distant from the parallel ends, and this sum, multiplied by $\frac{1}{6}$ of the height, will give the solidity.

Ex. 1. What is the solidity of a rectangular prismoid, the length and breadth of one end being 14 by 12 inches, and the other 6 by 4 inches, and the perpendicular 30 feet 6 inches.



First, $14\times12=168=$area of lower base.
$6\times 4=\ \ 24=$ " upper "

$\overline{192}$

$\left.\begin{array}{l}14+6\div2=10\\12+4\div2=\ 8\end{array}\right\}=\left\{\begin{array}{l}\text{length and breadth}\\\text{of middle section.}\end{array}\right.$

80
4
$\overline{320}=$area of 4 times mid. sec.

Then, 192
320
$\overline{512}$
$61=\frac{1}{6}$ height.
$\overline{512}$
3072
$\overline{31232}$

And, $31232\div1728=18.074$ *cubic ft. Ans.*

Ex. 2. Required the solidity of a stick of hewn timber whose lower end is 30 inches by 27, and whose upper end is 24 by 18 inches, supposing its height to be 48 feet?

Ans. 204 *cubic ft.*

Ex. 3. What weight of water can be put into a prismoidal vessel whose dimensions are as follows: at the top 5 feet by 3 feet, and at the bottom, 7 feet by 6 feet, and the perpendicular depth 4 feet, allowing $62\frac{1}{2}$ pounds avoirdupois to the cubic foot? *Ans.* 3375 *pounds.*

NOTE.—The weight of water which a vessel of any given dimensions will contain, may be found by calculating the capacity in cubic feet, and multiplying by $62\frac{1}{2}$ lbs. or 1000 ounces avoirdupois, the true weight of a cubic foot of pure water.

PROMISCUOUS EXAMPLES.

1. What is the whole area of a regular triangular prism, whose perimeter is 12 feet, and whose height is 22 feet?
Ans. ——

2. What must be paid for lining a rectangular cistern with lead, a 2*d* a pound, the thickness of the lead being such as to require 7 lbs. for each square foot of surface, the inner dimensions of the cistern being as follows; viz: the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches? *Ans.* ——

3. What is the solidity of a pentagonal pyramid, whose height is 16 feet, and the sides of whose base are each 3 feet 6 inches? *Ans.* ——

4. What is the lateral surface of a triangular pyramid whose slant height is 24 feet, and each side of whose base is 4 feet? *Ans.* ——

5. If the height of a frustrum of a pyramid be 30 feet, and the areas of the two ends 480 and 136, what is the solidity? *Ans.* ——

6. What are the solid contents of a triangular pyramid, whose height is 28 feet, and the sides of whose base are each 4 feet 7 inches? *Ans.* ——

7. Required the solidity of a wedge whose base is 4 feet 6 inches long by 2 feet 4 inches wide, the length of the edge being 5 feet, and the perpendicular height 12 feet?
Ans. ——

SECTION IV.

MEASURES OF THE FIVE REGULAR BODIES.

DEFINITIONS.

ART. 46.—When a body is contained under a certain number of similar and equal plane figures, it is called a *Regular* body.

Of this description are the following figures:

1. *The Tetraedron*, which has four equilateral triangular faces.

2. *The Hexaedron or Cube*, which has six square faces.

3. *The Octaedron*, whose sides are eight triangles.

4. *The Dodecaedron*, whose sides are twelve pentagons.

5. *The Icosaedron*, which has twenty equilateral triangular faces.

These comprise *all the regular solids*. They are formed by triangles, squares, and pentagons.

PROBLEM I.

To find the Convex Surface of a Regular Solid.

ART. 47. *Rule.*—Find the area of one of the sides, then multiply it by the number of sides, and the product will be the area. Or,

Multiply the proper tabular area or surface (taken from the following table, Prob. 2,) by the square of the linear edge of the solid, and the product will be the surface.

Since all the sides of a regular body are *equal*, it is manifest that the convex surface of any one of them multiplied by the number of sides will give the whole surface. Now, to facilitate the measurement of the surface of any regular body, a *Table* may be prepared containing the surfaces of the several regular solids, whose linear *edges* are *unity*. To construct such a *table*, we need only multiply the area of one of the sides as is given in Art. 13, by the number of sides. Thus the area of an equilateral triangle, whose edge is 1, is 0.4330127. Consequently the area

Of a square, $=.4330127\times4=$ 1.7320508.
Of a hexagon, $=.4330127\times6=$ 2.5980702.
Of a regular icosaedron, $=.4330127\times20=8.6602540$.

Ex. 1. What is the convex surface of a regular icosaedron, whose edges are each 3 feet?

OPERATION.

$3^2=9$, and $9\times8.6602540=77.9422860$, *Ans.*

Note.—This question is solved by multiplying the area of one side by the number of sides. Thus, the area of an equilateral triangle, whose edge is $1=.4330127$. Now, since there are 20 sides, if we multiply this number by 20, the product will be 8.6602540; which, multiplied by the square of one of the sides, gives the whole area as we see in the operation.

Ex. 2. Required the surface of a regular dodecaedron whose edges are each 25 inches. *Ans.* 89.6 *sq. ft.*

Ex. 3. What is the surface of a regular octaedron whose edges are each 76? *Ans.* 20008.63.

PROBLEM II.

To find the Solidity of any Regular Solid.

Art. 48. *Rule.*—I. Find the convex surface of the given solid by the previous rule.

II. Multiply the surface by $\frac{1}{3}$ of the perpendicular distance from the centre to one of the sides. Or,

Multiply the tabular solidity (taken from the following table) in the last column of the table by the cube of the linear edge, and the product will be the solid contents.

A Table of Surfaces and Solidities of Regular Bodies, the side being unity, or 1.

Number of sides.	Names.	Surfaces.	Solidity.
4	Tetraedron,	1.7320508	0.1178513
6	Hexaedron,	6.0000000	1.0000000
8	Octaedron,	3.4641016	0.4714045
12	Dodecaedron,	20.6457288	7.6631189
20	Icosaedron,	8.6602540	2.1816950

The above table may be used to great advantage in the measurement of other similar solids.

Ex. 1. What is the solidity of an octaedron, whose linear edge is 6 feet?

OPERATION.

First, $6^3=216$

Then, $216\times.4714045=101.8233$ *cubic feet.*

Ex. 2. What is the solidity of a regular octaedron, whose linear edges are each 32 inches?

Ans. 15447 *inches.*

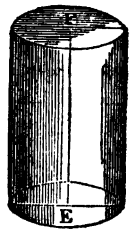
Ex. 3. What is the solidity of a regular hexaedron, whose linear edges are each 4 feet? *Ans.* 64

SECTION V.

MENSURATION OF THE CYLINDER, CONE AND SPHERE.

DEFINITIONS.

1



ART. 49. 1. A *Right Cylinder* is a solid, having equal and parallel circles for its ends, and is described by the revolution of a rectangle about one of its sides. Thus, *EF* (fig. 1) is a right cylinder.

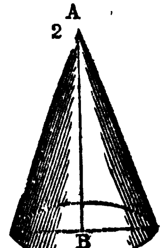
2. The *Axis* of a cylinder is a line passing through the centre, and is perpendicular to the bases; as, *EF* (fig. 1.)

3. The *Height* of a cylinder is the perpendicular distance from one base to the plane of the other.

4. If the cylinder be *oblique*, then the ends are equal and parallel circles, but inclined towards the axis.

NOTE.—A cylinder may be considered as a prism of an infinite number of sides; for the difference between such a prism and a right cylinder, would be less than any given quantity.

2



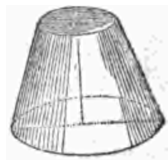
5. A *Right Cone* is a solid body of a true taper from the base to a point which is called the vertex, and is described by the revolution of a right-angled triangle about one of the sides which contains the right angle; as, *AB* (fig. 2.) The circle described by the revolving side is called the *base*, which is perpendicular to the axis that proceeds from the middle of the base to the vertex,

6. The *height* of a cone is the fixed side of the triangle by which it is described, or the perpendicular distance from the vertex to the plane of the base; as, *AB* (fig. 2.)

7. The *slant height* of a right cone is the distance from the vertex to the circumference of the base.

8. A *Frustrum* of a cone is what remains after a portion is cut off by a plane, parallel to the base: thus, fig. 3 exhibits the frustrum of a cone. The *height* of a frustrum is the perpendicular distance between its two parallel ends.

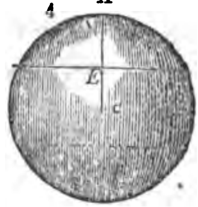
3



9. The *slant height* of the frustrum of a right cone is the distance between the peripheries of the two ends, measured upon the surface.

10. A *Sphere* is a solid, terminated by a curved surface, all the points of which are equally distant from a point within, called the centre. A sphere may be described by the revolution of a semi-circle about a diameter.

4



11. A *radius* of a sphere is a line drawn from the centre to any part of the surface; as, *CA* (fig. 4.)

12. The *diameter* of a sphere is a line drawn through the centre, and terminated at both ends by the surface. All diameters of a sphere are equal to each other, and each is double the radius.

13. A *Segment* of a sphere is a portion of the sphere cut off by any plane. *AB* (fig. 5) is a segment of a sphere. This plane is called the *base* of the segment.

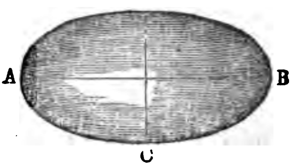
5

14. The *height* of a segment is the distance from the middle of its base to the convex surface.

15. A *Zone* is a portion of the surface of a sphere, included between two parallel planes which form its bases. If the bases are equally distant from the centre, it is called the *middle zone*.

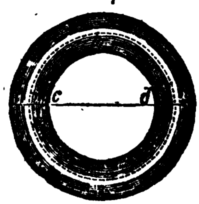
16. The *height* of a zone is the perpendicular distance between the two planes which form its bases.

6



17. A *Spheroid* is a solid, generated by the revolution of an ellipse about either of its axes; as, *AB* or *CD* (fig. 6.)

7



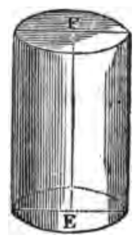
18. A *Cylindrical Ring* is a solid formed by bending a cylinder, as a cylindrical bar of iron, until the two ends meet each other; thus, *m,o,n,* (fig. 7) is a cylindrical ring.

PROBLEM I.

To find the Convex Surface of a Cylinder.

ART. 50. *Rule.*—Multiply the circumference of the base by the length of the cylinder, and the product will be the convex surface required: To this add the areas of the two ends when the entire surface is required.

The truth of this rule will easily appear by covering a cylinder with a paper, and then spreading it out into a plane. This plane will form a *parallelogram*, whose length is the same as the length of the cylinder, and whose breadth is the same as the perimeter or circumference of the cylinder. And by Art. 4, the area of the parallelogram thus formed, must be equal to the length multiplied into the breadth.



Ex. 1. What is the convex surface of a right cylinder, whose length is 23 feet, and the diameter of its base 3 feet?

OPERATION.

First, by Art. 16 find circumf. of base.
$3 \times 3.14159 = 9.42477$.
Then, $9.42477 \times 23 = 216.76971$ = surface.

Ex. 2. Required the entire surface of a cylinder whose length is 20 feet, and the diameter of whose base is 2 feet?

Ans. 131.95 *ft.*

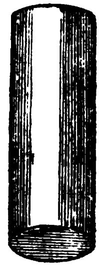
Ex. 3. What is the entire surface of a cylinder whose length is 82, and circumference of the base 71?

Ans. 6624.32.

PROBLEM II.

To find the Solidity of a Cylinder.

ART. 51. *Rule.*—Multiply the area of the base by the height, and the product will give the solid contents.



Ex. 1. What is the solidity of a cylinder, the diameter of whose base is 16 feet, and its height 28 feet ?

OPERATION.

First, find the area of the base by Art. 21.

$\overline{16}^2=256$. Then, $256\times.7854=201.0624=$ area of the base.

Then, $201.0624\times28=5629.7472=$solid contents.

Ex. 2. What is the solidity of a cylinder, whose height is 424, and the circumference of its base 213 ?

Ans. 1530837.

Ex. 3. What is the solidity of a cylinder whose length is 5 feet, and the diameter of the end 2 feet ?

Ans. 15.780 *solid ft.*

Ex. 4. Required the solidity of a cylinder the circumference of whose base is 40 feet, and the height 20 feet.

Ans. 254.656. *solid ft.*

Ex. 5. The Winchester bushel is a hollow cylinder, $18\frac{1}{2}$ inches in diameter and 8 inches deep : what is its capacity ?

First, the area of the base$=\overline{18.5}^2\times.7854=268.8025$.

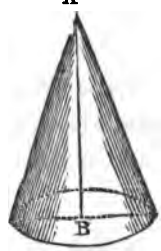
Then, $268.8025\times8=2150.42.00=$capacity in cubic inches.

NOTE.—By this rule, every sealer of Weights and Measures may determine the exact capacity of any *measure* submitted to his inspection. And so any one may test the accuracy of any measure, whether dry or liquid, by reducing its capacity to cubic inches and dividing by the number of cubic inches contained in such measure. The divisor for any measure may be found in the Table of Weights and Measure, page 9.

PROBLEM III.

To find the Convex Surface of a Cone.

ART. 52. *Rule.*—Multiply the perimeter of the base by the slant height, and $\frac{1}{2}$ the product will be the surface, to which add the area of the base when the entire surface is required.



Ex. 1. The diameter of the base of a right cone is 3 feet, and the slant height is 15 feet; what is the convex surface?

OPERATION.

First, $3 \times 3.14159 = 9.42477$ = circ. of base.
Then, $9.42477 \times 15 \div 2 = 70.686$ sq. ft.

Ex. 2. The diameter of the base of a cone is 4.5 feet, and the slant height 20 feet; what is the entire surface?

Ans. 157.276 *sq. ft.*

Ex. 3. If the axis of a cone be 16 and the circumference of the base 75.4, what is the whole surface?

First, find the diameter, $\frac{1}{2}$ of which will be the *leg* of the triangle; then the square root of the sum of the squares of the perpendicular and base will be the slant height?

Ans. 1206.4.

Ex 4. The circumference of the base of a cone is 10.75, and the slant height 18.25; what is the surface?

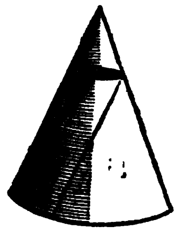
Ans. 98.0937.

PROBLEM IV.

To find the Solidity of a Cone.

ART. 53. *Rule.*—Multiply the area of the base by $\frac{1}{3}$ of the height, and the product will be the solidity.

We have already seen that the solidity of the cylinder is equal to the product of the area of the base into the perpendicular height, (Art. 51.) Now, it may be proved by Geometry, that if a cone and a cylinder have the same base and altitude, the cone is $\frac{1}{3}$ of the cylinder. Consequently, the solidity of the cone is equal to the area of the base into $\frac{1}{3}$ of the height.



Ex. 1. What is the solidity of a right cone whose height is $10\frac{1}{2}$ feet, and the circumference of the base is 9 feet?

We here multiply the area of the base by $\frac{1}{3}$ of the height, and the product is the solidity.

OPERATION.

First, $9^2=81$, and $10\frac{1}{2}\div 3=3\frac{1}{2}=\frac{1}{3}$ height.
Now, $81\times .7854=63.6174$, area of base.
Then, $63.6174\times 3\frac{1}{2}=222.6609$, *Ans.*

Ex. 2. What is the solidity of a cone whose base is 3 feet 6 inches in diameter, and the perpendicular height 9 feet? *Ans.* 28.86345 *cubic ft.*

Ex. 3. Required the solidity of a cone whose height is 663, and the diameter of whose base is 101? *Ans.* 1770622.

Ex. 4. What is the solidity of a cone the circumference of whose base is 40 feet, and the height 50 feet? *Ans.* 2122.1333 *solid ft.*

PROBLEM V.

To find the Surface of a Frustrum of a Cone.

ART. 54. *Rule.*—Add together the circumferences of the two ends, and multiply the sum by $\frac{1}{2}$ the slant height of the

frustrum; the product will be the convex surface: to which add the areas of the two bases when the entire surface is required.

This rule is precisely the same as that for a *frustrum* of a pyramid (Art. 41;) and if a cone be considered as a pyramid of an infinite number of sides, it is equally applicable to the measurement of the *frustrum* of a cone.

Ex. 1. What is the convex surface of the frustrum of a cone, the circumference of the greater base being 30 feet, and of the smaller, 10 feet, the slant height being 20 feet?

OPERATION.

$$\begin{array}{rl} & 30 \\ & \underline{10} \\ & 40 = \text{circum. of the two ends.} \\ \text{half slant height} = & \underline{10} \\ & 400 \text{ sq. ft.} \end{array}$$

Ex. 2. Required the convex surface of the frustrum of a cone, the diameter of the greater base being 44, and of the smaller 33, and the slant height 84? *Ans.* 10159.8.

Ex. 3. What is the entire surface of the frustrum of a cone whose slant height is 20 feet, and the diameters of the bases 8 and 4 feet? *Ans.* 439.824 *sq. ft.*

PROBLEM VI.

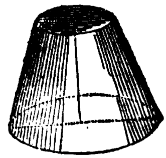
To find the Solidity of the Frustrum of a Cone.

Art. 55. *Rule.*—I. Add to the areas of the two ends of the frustrum the *square root of their product.*

II. Multiply this sum by $\frac{1}{3}$ of the perpendicular height, and the product will be the solidity.

If a *cone* and a *pyramid* have equal bases and altitudes, they are equal in their solidity. Consequently, the rule already given for the *frustrum* of a *pyramid* is equally applicable to the frustrum of a cone. (Art. 41.)

Ex. 1. How many cubic feet in a piece of round timber, the diameter of the greater end being 18 inches, and that of the smaller, 9 inches, and the length 14.25 feet?



OPERATION.

First, $\overline{18}^2=324$, and $324\times.7854=254.4696$

$\overline{9}^2=81$, and $81\times.7854=63.6174$

$318.0870 = \left\{\begin{array}{l}\text{sum of areas}\\\text{of two ends.}\end{array}\right.$

Then, $\sqrt{254.4696\times63.6174}=127.23$=square root of product of the two areas.

And, 254.4696
63.6174
127.234

$445.3210 = \left\{\begin{array}{l}\text{sum of areas of two ends in inches,}\\\text{and square root of their product.}\end{array}\right.$

Then, 14.25 feet=171 inches.

Therefore, $445.3210\times171\div1728=14.6894$ solid feet.

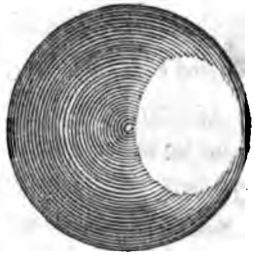
Ex. 2. How many gallons of ale are contained in a cistern in the form of a conic frustrum, if the larger diameter be 9 feet, and the smaller diameter 7 feet, and the depth 9 feet?

PROBLEM VII.

To find the Surface of a Sphere or Globe.

ART. 56. *Rule.*—I. Multiply the diameter of the sphere by its circumference, and the product will be the surface. Or,

II. Multiply the square of the diameter by 3.14159.



Ex. 1. What is the surface of a sphere whose diameter is 7 feet?

OPERATION.

First, $7 \times 3.14159 = 21.99113 =$ circumference.

Then, $21.99113 \times 7 = 153.93791$ sq. ft. = surface.

Ex. 2. The dome of the Capitol at Washington is hemispherical, and is 95 feet across the base; how many square feet in its convex surface? *Ans.* 28352.85 *sq. ft.*

NOTE.—By the same rule the mechanic may readily determine the number of square feet of sheathing, painting, wainscoting, or plastering, in the *external* or *internal* surface of any dome, however large or small. If the dome be not hemispherical, it is the *segment* or zone of a sphere, and its convex surface is found by Art. 57, following.

Ex. 3. How many square feet of lead will it require to cover a hemispherical dome whose base is 13 feet across? *Ans.* 265.5.

NOTE.—The *surface* of a sphere is equal to the convex surface of a cylinder whose diameter is equal to the diameter of the sphere, and whose height is also equal to the altitude of the sphere. Or, the surface of a sphere is equal to 4 times the area of a circle of the same diameter. Therefore, to find the surface of a sphere, we have the following concise Rule: *Multiply the area of a circle of the same diameter of the sphere into* 4, *and the product will be the entire surface.* Consequently, the *area* of a *hemisphere* is equal to *twice* the area of the base. For the area of a circle is equal to the product of half the diameter into half the circumference, or, what is the same in the result, $\frac{1}{4}$ of the product of the diameter and circumference. Therefore, if the circumference of a circle be multiplied by its diameter, the product will be 4 *times* the area of the circle.

PROBLEM VIII.

To find the Convex surface of a Spherical Zone or Segment.

Art. 57. *Rule.*—Multiply the height of the zone or segment by the whole circumference of the sphere of which it is a part, and the product will be the convex surface.

Note.—The convex surface of any spherical segment or zone is equal to that of the circumscribed cylinder, by Art. 58, (Note.)

Ex. 1. If the axis of a sphere, be 42 inches, what is the convex surface of a segment or zone *ABD*, whose height *ED* is 9 inches?

OPERATION.

First, $42 \times 3.14159 = 131.9468$=circumference.
9=height.

1187.5212=surface in square inches.

Ex. 2. If the diameter of the earth be 7930 miles, what is the convex surface of the torrid zone extending 23°20′ on each side of the equator.

Ans. 78.669.700.*sq. miles.*

Ex. 3. The diameter of a sphere is 25 feet and the height of the zone is 4 feet, what is the convex surface of the zone?

Ans. 314.159.

PROBLEM IX.

To find the Solidity of a Sphere or Globe.

Art. 58. *Rule* I.—Multiply the surface by the diameter, and $\frac{1}{6}$ of the product will be the solidity. Or,

II. Multiply the square of the diameter by the circumference, and $\frac{1}{6}$ of the product will be the contents. Or,

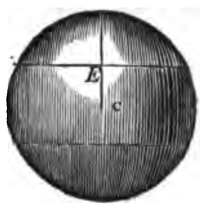
III. Multiply the cube of the diameter by the decimal .5236.

NOTE.—If we put D=the diameter, C=circumference, and S=the surface of the sphere, or its equal the circumscribing cylinder; also, A=the number 3.14159; Then $\frac{1}{4}$ of S is=the base of the cylinder, and D is the height of the cylinder; therefore, $\frac{1}{4}\ D\times S$ is the solidity of the cylinder. But it is proved that a *sphere* is *two thirds* of its circumscribing cylinder (Sup. Euc. 21.3.) Therefore, $\frac{2}{3}$ of $\frac{1}{4}\ D\times S$, or its equal, $\frac{1}{6}D\times S$=the solidity of the sphere according to the first rule.

2. But since the solidity of the cylinder is equal to its height multiplied into the area of its base, (Art. 51,) and since the height and diameter of the cylinder are each equal to the diameter of the sphere, if we put D again for the diameter, we shall have by the 3d rule: $D^2\times .7854\times D$, or $D^3\times .7854$=solidity of circumscribing cylinder. And since the solidity of the *sphere* is $\frac{2}{3}$ of this, we have $D^3\times .5236$=the solidity of the sphere.

3. There are certain numbers frequently occurring in mathematical investigations, and particularly in determining the areas of circles and solidities of spheres, &c., which cannot be made too familiar to the mind. They are the following: 3.14159; .7854; and .5236. The first represents the ratio of the *circumference* of a circle to its diameter; the second expresses the ratio of the area of a circle to the square of the diameter; and the third, the ratio of the solidity of a sphere to the cube of its diameter.

If we divide 3.14159 by 4, the quotient is .7854 very nearly, and again 3.14159 divided by 6=.5236.



Ex. 1. What is the solidity of a globe whose diameter is 12 inches?

OPERATION.

$\overline{12}^2\times 3.14159=452.38996$=surface of the sphere.
Then, $452.38996\times 12\div 6=904.78$=solidity.

Or thus: $\overline{12}^3=1728$=cube of the diameter.
And, $1728\times .5236=904.78$=solid contents.

Ex. 2. What is the solidity of the earth, if it be a sphere, 7930 miles diameter?

Ans. 261.107.000.000. *cubic miles.*

Ex. 3. The diameter of a sphere is 16; what is its solidity? *Ans.* 2144.6656.

Ex. 4. Required the solidity of a globe whose diameter is $18\frac{1}{2}$ inches. *Ans.* 3315.23 *cubic inches.*

Art. 59.—Knowing the *solidity* of a sphere, we may determine the diameter by reversing the third rule in this article; that is, by *dividing the solidity by* .5236 and *extracting the cube root of the quotient.*

Ex. 1. What is the diameter of a sphere whose solidity is 4.188.8000.

In this case we divide the solidity by .5236, and extract the cube root of the quotient for the diameter; and so the diameter of any solid may be found, whose capacity is given.

OPERATION.

```
.5236)4188.8000(8000
      4188.8
      ------
           0
```

And, $\sqrt[3]{8000}=20=$diameter.

Ex. 2. Required the diameter of a sphere whose solidity is 113.0976. *Ans.* 6.

Ex 3. What must be the diameter of a globe to contain 16755 pounds of water? *Ans.* 8 *ft.*

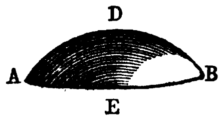
PROBLEM X.

To find the Solidity of a Spherical Segment.

Art. 60. *Rule.*—To three times the square of the radius

of its base, add the square of its height; then multiply the sum by the height, and the product by .5236, for the contents.

Ex. 1. What is the solidity of the segment *ABD*, the height *ED* being 4 feet, and the diameter of the base *AB* being 14 feet?



OPERATION.

First, $7^2 \times 3 + 4^2 = 147 + 16 = 163$

Then, $163 \times 4 \times .5236 = 341.3872$ solid feet.

Ex. 2. If the height of a spherical segment be 8 feet, and the diameter of its base 25 feet, what is the solidity?

Ans. 2231.5832.

Ex. 3. Required the solidity of a spherical segment whose height is 6 and the radius of its base 12?

Ans. 1470.27.

Art. 61.—The solidity of a spherical segment is frequently required when the radius of its base is not given, but if the *diameter* of the sphere and the height of the segment be known, the solidity may be easily found by the following

Rule.—From three times the diameter of the sphere, subtract twice the height of the segment; then multiply the remainder by the square of the height and the product by the decimal .5236.

Ex. 1. What is the solidity of a spherical segment, whose height is 2 feet, cut from a sphere known to be 8 feet in diameter?

OPERATION.

First, $8\times3=24=24$
$\quad\quad 4$=square of height.
$\quad\quad \overline{20}$
$\quad\quad 4$= " "
$\quad\quad \overline{80}$

Then, $80\times.5236=41.8880$, *Ans.*

Ex. 2. What is the solidity of a spherical segment, whose height is 3 feet, cut from a sphere whose diameter is 18 feet? *Ans.* 226.1952.

PROBLEM XI.

To find the Solidity of a Spheroid.

Art. 62. *Rule.*—Multiply the square of the revolving axis by the fixed axis; and the product multiplied by .5236 will give the solidity.

Ex. 1. What is the solidity of an oblong spheroid, whose longer axis is 30, and the shorter, 20, the revolving axis being the shorter?

Note.—If the generating ellipse revolves about its major axis, the spheroid is *prolate* or oblong; if about its minor axis, the spheroid is *oblate*.

OPERATION.

$\overline{20}^2\times30=12000$.

Then, $12000\times.5236=6283.2000$

Ex. 1. What is the solidity of a prolate spheroid whose fixed axis is 100, and revolving axis 6 feet?

Ans. 1884.96.

Ex. 2. What is the solidity of an oblate spheroid whose axes are 20 and 10? *Ans.* 2094.4.

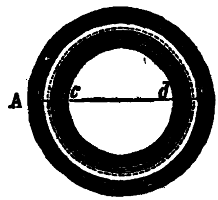
Ex. 3. What is the solidity of an oblate spheroid, whose axes are 36 and 28? *Ans.* 19010.3968.

PROBLEM XII.

To find the Convex Surface of a Cylindrical Ring.

ART. 63. *Rule.*—To the thickness of the ring, add the inner diameter; then multiply this sum by the thickness, and the product by 9.8696 (which is the square of 3.1416) and it will give the convex surface required.

Ex. 1. The thickness *Ac* of a cylyndrical ring is 4 inches, and the inner diameter *cd* is 14 inches; required the convex surface.



OPERATION.

$Ac+cd=4+14=18$

Then, $18\times4\times9.8696=710.612$ sq. in.=convex surface.

Ex. 2. The thickness of a cylindrical ring is 3 inches, and the inner diameter 12 inches, what is the convex surface? *Ans.* 444.132 *sq. inches.*

Ex. 3. The thickness of a cylindrical ring is 6 inches, and the inner diameter 20 inches, required the convex surface? *Ans.* 1539.6576. *sq. inches.*

PROBLEM XIII.

To find the Solidity of a Cylindrical Ring.

ART. 64. *Rule.*—To the thickness of the ring, add the inner diameter; then multiply the sum by the square of the thickness, and the product by 2.4674 (which is $\frac{1}{4}$ of the square of 3.1416) and it will give the solidity.

Ex. 1. Required the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches.

OPERATION.

First, 3+8=11

$\underline{\quad 9}$=sq. of thickness.

99×2.4674=244.2726=solidity in inches.

Ex. 2. The inner diameter of a cylindrical ring, is 14 inches, and the thickness in metal 4 inches, what is the solidity of the ring? *Ans.* 710.612 *solid inches.*

Ex. 3. Required the solidity of a cylindrical ring, whose thickness is $8\frac{1}{2}$ inches, and inner diameter $22\frac{1}{2}$ inches?

Ans. ——

Ex. 4. What is the solidity of a cylindrical ring, whose thickness is $4\frac{1}{2}$ inches, and inner diameter $5\frac{3}{8}$ inches?

Ans. ——

Ex. 5. What is the solidity of a cylindrical ring, whose thickness is $1\frac{3}{8}$ inches, and inner diameter 15 inches?

Ans. ——

PROMISCUOUS EXAMPLES.

1. What is the solidity of the greatest square prism which can be cut from a cylindrical stick of timber 2 feet 6 inches diameter, and 56 feet long? *Ans.* 175 *cubic ft.*

2. How much water can be put into a cubical vessel three feet deep, which has been previously filled with cannon balls of the same size, 2. 4. 6 or 9 inches in diameter, regularly arranged in tiers one directly above another?

Ans. $96\frac{1}{2}$ *wine gallons.*

3. What will be the expense of painting a conical spire, at 8 cents per square yard, whose height is 120 feet and circumference at the base 60 feet? *Ans.* $32.

4. What is the solidity of a conic frustrum whose height is 40 feet, the greater diameter 18 feet, and the smaller, 9 feet? *Ans.* ——

5. What is the solidity of the greatest cube which can be cut from a sphere three feet in diameter?

Ans. $5\frac{1}{5}$ *ft.*

6. What is the solidity of a cylinder whose height is 20 feet, and the circumference of the base 20 feet?

Ans. 636.64 *cubic ft.*

7. What is the solidity of a spherical segment whose height is 26 feet, and the diameter of the base 48 feet?

Ans. ——

8. How many such globes as the earth are equal in bulk to the sun, if the former is 7930 miles in diameter, and the latter 890,000? *Ans.* 1.413.678.

9. The diameter of a sphere is 12 feet; required its solidity? *Ans.* ——

10. What is the solidity of a cylindrical ring whose thickness is 8 inches, and inner diameter 19 inches?

Ans. ——

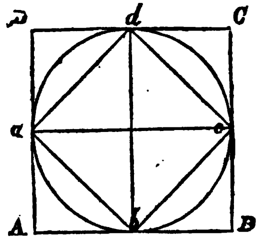
11. What is the solidity of a *prolate* spheroid, whose axes are 55 and 33? *Ans.* 31361.022.

12. What is the solidity of an *oblate* spheroid whose axes are 67 and 48 feet? *Ans.* ——

NOTE TO PROBLEM VIII. PAGE 50.

The *area* of a circle is to the area of the *circumscribed square* as .7954 is to 1, and to that of the *inscribed square* as .7854 is to $\frac{1}{2}$. Consequently the square described within a circle is precisely half of the square described without it.

To illustrate this proposition let *abcd* in the annexed figure be the inscribed square, and *ABCD* the circumscribed square of the circle *abcd*.



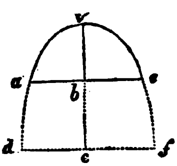
The area of the circle is equal to $\overline{ac}^2 \times .7854$ (Art. 19.) But the area of the circumscribed square (Art. 4.) is equal to $\overline{AB}^2$; or the side $AB \times BC$. And we see that the inscribed square contains only 4 equal right angled triangles, while the circumscribed square contains 8.

Ex. 1. The circumference of a circle is 715, what is the side of the inscribed square? (Consult Art. 20 and 24.)

Ans. 160.9465.

NOTE TO PROBLEM XIX. PAGE 59.

A much shorter method for finding the area of a Hyperbola is the following:



To find the area of a Hyperbola.

Rule.—Multiply the base by $\frac{2}{3}$ of the height and subtract $\frac{1}{19.2}$ of the product; the remainder will be the area, very nearly.

Ex. 1. If the base of a hyperbola be 24 feet, the height 10 feet, and the transverse axis 30; what is the area?

OPERATION.

$24 \times 6\frac{2}{3} = 160$. Then, $160 - 8.33 = 151.67$ *Ans.*

TABLES OF SQUARES AND CUBES,

To facilitate the Mensuration of the Surfaces and Solidities of Bodies.

Number.	Square.	Cube.	Number.	Square.	Cube.
1	1	1	50	2500	125000
2	4	8	51	2601	132651
3	9	27	52	2704	140608
4	16	64	53	2809	148877
5	25	125	54	2916	157464
6	36	216	55	3025	166375
7	49	343	56	3136	175616
8	64	512	57	3249	185193
9	81	729	58	3364	195112
10	100	1000	59	3481	205379
11	121	1331	60	3600	216000
12	144	1728	61	3721	226981
13	169	2197	62	3844	238328
14	196	2744	63	3969	250047
15	225	3375	64	4096	262144
16	256	4096	65	4225	274625
17	289	4913	66	4356	287496
18	324	5832	67	4489	300763
19	361	6859	68	4624	314432
20	400	8000	69	4761	328509
21	441	9261	70	4900	343000
22	484	10648	71	5041	357911
23	529	12167	72	5184	373248
24	576	13824	73	5329	389017
25	625	15625	74	5476	405224
26	676	17576	75	5625	421875
27	729	19683	76	5776	438976
28	784	21952	77	5929	456533
29	841	24389	78	6084	474552
30	900	27000	79	6241	493039
31	961	29791	80	6400	512000
32	1024	32768	81	6561	531441
33	1089	35937	82	6724	551368
34	1156	39304	83	6889	571787
35	1225	42875	84	7056	592704
36	1296	46656	85	7225	614125
37	1369	50653	86	7396	636056
38	1444	54872	87	7569	658503
39	1521	59319	88	7744	681472
40	1600	64000	89	7921	704969
41	1681	68921	90	8100	729000
42	1764	74088	91	8281	753571
43	1849	79507	92	8464	778688
44	1936	85184	93	8649	804357
45	2025	91125	94	8836	830584
46	2116	97336	95	9025	857375
47	2209	103823	96	9216	884736
48	2304	110592	97	9409	912673
49	2401	117649	98	9604	941192

Number.	Square.	Cube.	Number.	Square.	Cube.
99	9801	970299	150	22500	3375000
100	10000	1000000	151	22801	3442951
101	10201	1030301	152	23104	3511808
102	10404	1061208	153	23409	3581577
103	10609	1092727	154	23716	3652264
104	10816	1124864	155	24025	3723875
105	11025	1157625	156	24336	3796416
106	11236	1191016	157	24649	3869893
107	11449	1225043	158	24964	3944312
108	11664	1259712	159	25281	4019679
109	11881	1295029	160	25600	4096000
110	12100	1331000	161	25921	4173281
111	12321	1367631	162	26244	4251528
112	12544	1404928	163	26569	4330747
113	12769	1442897	164	26896	4410944
114	12996	1481544	165	27225	4492125
115	13225	1520875	166	27556	4574296
116	13456	1560896	167	27889	4657463
117	13689	1601613	168	28224	4741632
118	13924	1643032	169	28561	4826809
119	14161	1685159	170	28900	4913000
120	14400	1728000	171	29241	5000211
121	14641	1771561	172	29584	5088448
122	14884	1815848	173	29929	5177717
123	15129	1860867	174	30276	5268024
124	15376	1906624	175	30625	5359375
125	15625	1953125	176	30976	5451776
126	15876	2000376	177	31329	5545233
127	16129	2048383	178	31684	5639752
128	16384	2097152	179	32041	5735339
129	16641	2146689	180	32400	5832000
130	16900	2197000	181	32761	5929741
131	17161	2248091	182	33124	6028568
132	17424	2299968	183	33489	6128487
133	17689	2352637	184	33856	6229504
134	17956	2406104	185	34225	6331625
135	18225	2460375	186	34596	6434856
136	18496	2515456	187	34969	6539203
137	18769	2571353	188	35344	6644672
138	19044	2628072	189	35721	6751269
139	19321	2685619	190	36100	6859000
140	19600	2744000	191	36481	6967871
141	19881	2803221	192	36864	7077888
142	20164	2863288	193	37249	7189057
143	20449	2924207	194	37636	7301384
144	20736	2985984	195	38025	7414875
145	21025	3048625	196	38416	7529536
146	21316	3112136	197	38809	7645373
147	21609	3176523	198	39204	7762392
148	21904	3241792	199	39601	7880599
149	22201	3307949	200	40000	8000000

Number.	Square.	Cube.	Number.	Square.	Cube.
201	40401	8120601	251	63001	15813251
202	40804	8242408	252	63504	16003008
203	41209	8365427	253	64009	16194277
204	41616	8489664	254	64516	16387064
205	42025	8615125	255	65025	16581375
206	42436	8741816	256	65536	16777216
207	42849	8869743	257	66049	16974593
208	43264	8998912	258	66564	17173512
209	43681	9123329	259	67081	17373979
210	44100	9261000	260	67600	17576000
211	44521	9393931	261	68121	17779581
212	44944	9528128	262	68644	17984728
213	45369	9663597	263	69169	18191447
214	45796	9800344	264	69696	18399744
215	46225	9938375	265	70225	18609625
216	46656	10077696	266	70756	18821096
217	47089	10218313	267	71289	19034163
218	47524	10360232	268	71824	19248832
219	47961	10503459	269	72361	19465109
220	48400	10648000	270	27900	19683000
221	48841	10793861	271	73441	19902511
222	49284	10941048	272	73984	20123648
223	49729	11089567	273	74529	20346417
224	50176	11239424	274	75076	20570824
225	50625	11390625	275	75625	20796875
226	51076	11543176	276	76176	21024576
227	51529	11697083	277	76729	21253933
228	51984	11852352	278	77284	21484952
229	52441	12008989	279	77841	21717639
230	52900	12167000	280	78400	21952000
231	53361	12326391	281	78961	22188041
232	53824	12487168	282	79524	22425768
233	54289	12649337	283	80089	22665187
234	54756	12812904	284	80656	22906304
235	55225	12977875	285	81225	23149125
236	55696	13144256	286	81796	23393656
237	56169	13312053	287	82369	23639903
238	56644	13481272	288	82944	23887872
239	57121	13651919	289	83521	24137569
240	57600	13824000	290	84100	24389000
241	58081	13997521	291	84681	24642171
242	58564	14172488	292	85264	24897088
243	59049	14348907	293	85849	25153757
244	59536	14526784	294	86436	25412184
245	60025	14706125	295	87025	25672375
246	60516	14886936	296	87616	25934336
247	61009	15069223	297	88209	26198073
248	61504	15252992	298	88804	26463592
249	62001	15438249	299	89401	26730899
250	62500	15625000	300	90000	27000000

Number.	Square.	Cube.	Number.	Square.	Cube.
301	90601	27270901	351	123201	43243551
302	91204	27543608	352	123904	43614208
303	91809	27818127	353	124609	43986977
304	92416	28094464	354	125316	44361864
305	93025	28372625	355	126025	44738875
306	93636	28652616	356	126736	45118016
307	94249	28934443	357	127449	45499293
308	94864	29218112	358	128164	45882712
309	95481	29503629	359	128881	46268279
310	96100	29791000	360	129600	46656000
311	96721	30080231	361	130321	47045881
312	97344	30371328	362	131044	47437928
313	97969	30664297	363	131769	47832147
314	98596	30959144	364	132496	48228544
315	99225	31255875	365	133225	48627125
316	99856	31554496	366	133956	49027896
317	100489	31855013	367	134689	49430863
318	101124	32157432	368	135424	49836032
319	101761	32461759	369	136161	50243409
320	102400	32768000	370	136900	50653000
321	103041	33076161	371	137641	51064811
322	103684	33386248	372	138384	51478848
323	104329	33698267	373	139129	51895117
324	104976	34012224	374	139876	52313624
325	105625	34328125	375	140625	52734375
326	106276	34645976	376	141376	53157376
327	106929	34965783	377	142129	53582633
328	107584	35287552	378	142884	54010152
329	108241	35611289	379	143641	54439939
330	108900	35937000	380	144400	54872000
331	109561	36264691	381	145161	55306341
332	110224	36594368	382	145924	55742968
333	110889	36926037	383	146689	56181887
334	111556	37259704	384	147456	56623104
335	112225	37595375	385	148225	57066625
336	112896	37933056	386	148996	57512456
337	113569	38272753	387	149769	57960603
338	114244	38614472	388	150544	58411072
339	114921	38958219	389	151321	58863869
340	115600	39304000	390	152100	59319000
341	116281	39651821	391	152881	59776471
342	116964	40001688	392	153664	60236288
343	117649	40353607	393	154449	60698457
344	118336	40707584	394	155236	61162984
345	119025	41063625	395	156025	61629875
346	119716	41421736	396	156816	62099136
347	120409	41781923	397	157609	62570773
348	121104	42144192	398	158404	63044792
349	121801	42508549	399	159201	63521199
350	122500	42875000	400	160000	64000000

Number.	Square.	Cube.	Number.	Square.	Cube.
401	160801	64481201	451	203401	91733851
402	161604	64964808	452	204304	92345408
403	162409	65450827	453	205209	92959677
404	163216	65939264	454	206116	93576664
405	164025	66430125	455	207025	94196375
406	164836	66923416	456	207936	94818816
407	165649	67419143	457	208849	95443993
408	166464	67917312	458	209764	96071912
409	167281	68417929	459	210681	96702579
410	168100	68921000	460	211600	97336000
411	168921	69426531	461	212521	97972181
412	169744	69934528	462	213444	98611128
413	170569	70444997	463	214369	99252847
414	171396	70951944	464	215296	99897344
415	172225	71473375	465	216225	100544625
416	173056	71991296	466	217156	101194696
417	173889	72511713	467	218089	101847563
418	174724	73034632	468	219024	102503232
419	175561	73560059	469	219961	103161709
420	176400	74088000	470	220900	103823000
421	177241	74618461	471	221841	104487111
422	178084	75151448	472	222784	105154048
423	178929	75686967	473	223729	105823817
424	179776	76225024	474	224676	106496424
425	180625	76765625	475	225625	107171875
426	181476	77308776	476	226576	107850176
427	182329	77854483	477	227529	108531333
428	183184	78402752	478	228484	109215352
429	184041	78953589	479	229441	109902239
430	184900	79507000	480	230400	110592000
431	185761	80062991	481	231361	111284641
432	186624	80621568	482	232324	111980168
433	187489	81182737	483	233289	112678587
434	188356	81746504	484	234256	113379904
435	189225	82312875	485	235225	114084125
436	190096	82881856	486	236196	114791256
437	190969	83453453	487	237169	115501303
438	191844	84027672	488	238144	116214272
439	192721	84604519	489	239121	116930169
440	193600	85184000	490	240100	117649000
441	194481	85766121	491	241081	118370771
442	195364	86350888	492	242064	119095488
443	196249	86938307	493	243049	119823157
444	197136	87528384	494	244036	120553784
445	198025	88121125	495	245025	121287375
446	198916	88716536	496	246016	122023936
447	199809	89314623	497	247009	122763473
448	200704	89915392	498	248004	123505992
449	201601	90518849	499	249001	124251499
450	202500	91125000	500	250000	125000000

Number.	Square.	Cube.
501	251001	125751501
502	252004	126506008
503	253009	127263527
504	254016	128024064
505	255025	128787625
506	256036	129554216
507	257049	130323843
508	258064	131096512
509	259081	131872229
510	260100	132651000
511	261121	133432831
512	262144	134217728
513	263169	135005697
514	264196	135796744
515	265225	136590875
516	266256	137388096
517	267289	138188413
518	268324	138991832
519	269361	139798359
520	270400	140608000
521	271441	141420761
522	272484	142236648
523	273529	143055667
524	274576	143877824
525	275625	144703125
526	276676	145531576
527	277729	146363183
528	278784	147197952
529	279841	148035889
530	280900	148877000
531	281961	149721291
532	283024	150568768
533	284089	151419437
534	285156	152273304
535	286225	153130375
536	287296	153990656
537	288369	154854153
538	289444	155720872
539	290521	156590819
540	291600	157464000
541	292681	158340421
542	293764	159220088
543	294849	160103007
544	295936	160989184
545	297025	161878625
546	298116	162771336
547	299209	163667323
548	300304	164566592
549	301401	165469149
550	302500	166375000
551	303601	167284151
552	304704	168196608
553	305809	169112377
554	306916	170031464
555	308025	170953875
556	309136	171879616
557	310249	172808693
558	311364	173741112
559	312481	174676879
560	313600	175616000
561	314721	176558481
562	315844	177504328
563	316969	178453547
564	318096	179406144
565	319225	180362125
566	320356	181321496
567	321489	182284263
568	322624	183250432
569	323761	184220009
570	324900	185193000
571	326041	186169411
572	327184	187149248
573	328329	188132517
574	329476	189119224
575	330625	190109375
576	331776	191102976
577	332929	192100033
578	334084	193100552
579	335241	194104539
580	336400	195112000
581	337561	196122941
582	338724	197137368
583	339889	198155287
584	341056	199176704
585	342225	200201625
586	343396	201230056
587	344569	202262003
588	345744	203297472
589	346921	204336469
590	348100	205379000
591	349281	206425071
592	350464	207474688
593	351649	208527857
594	352836	209584584
595	354025	210644875
596	355216	211708736
597	356409	212776173
598	357604	213847192
599	358801	214921799
600	360000	216000000

Number.	Square.	Cube.	Number.	Square.	Cube.
601	361201	217081801	651	423801	275894451
602	362404	218167208	652	425104	277167808
603	363609	219256227	653	426409	278445077
604	364816	220348864	654	427716	279726264
605	366025	221445125	655	429025	281011375
606	367236	222545016	656	430336	282300416
607	368449	223648543	657	431649	283593393
608	369664	224755712	658	432964	284890312
609	370881	225866529	659	434281	286191179
610	372100	226981000	660	435600	287496000
611	373321	228099131	661	436921	288804781
612	374544	229220928	662	438244	290117528
613	375769	230346397	663	439569	291434247
614	376996	231475544	664	440896	292754944
615	378225	232608375	665	442225	294079625
616	379456	233744896	666	443556	295408296
617	380689	234885113	667	444889	296740963
618	381924	236029032	668	446224	298077632
619	383161	237176659	669	447561	299418309
620	384400	238328000	670	448900	300763000
621	385641	239483061	671	450241	302111711
622	386884	240641848	672	451584	303464448
623	388129	241804367	673	452929	304821217
624	389376	242970624	674	454276	306182024
625	390625	244140625	675	455625	307546875
626	391876	245314376	676	456976	308915776
627	393129	246491883	677	458329	310288733
628	394384	247673152	678	459684	311665752
629	395641	248858189	679	461041	313046839
630	396900	250047000	680	462400	314432000
631	398161	251239591	681	463761	315821241
632	399424	252435968	682	465124	317214568
633	400689	253636137	683	466489	318611987
634	401956	254840104	684	467856	320013504
635	403225	256047875	685	469225	321419125
636	404496	257259456	686	470596	322828856
637	405769	258474853	687	471969	324242703
638	407044	259694072	688	473344	325660672
639	408321	260917119	689	474721	327082769
640	409600	262144000	690	476100	328509000
641	410881	263374721	691	477481	329939371
642	412164	264609288	692	478864	331373888
643	413449	265847707	693	480249	332812557
644	414736	267089984	694	481636	334255384
645	416025	268336125	695	483025	335702375
646	417316	269586136	696	484416	337153536
647	418609	270840023	697	485809	338608873
648	419904	272097792	698	487204	340068392
649	421201	273359449	699	488601	341532099
650	422500	274625000	700	490000	343000000

Number.	Square.	Cube.	Number.	Square.	Cube.
701	491401	344472101	751	564001	423564751
702	492804	345948408	752	565504	425259008
703	494209	347428927	753	567009	426957777
704	495616	348913664	754	568516	428661064
705	497025	350402625	755	570025	430368875
706	498436	351895816	756	571536	432081216
707	499849	353393243	757	573049	433798093
708	501264	354894912	758	574564	435519512
709	502681	356400829	759	576081	437245479
710	504100	357911000	760	577600	438976000
711	505521	359425431	761	579121	440711081
712	506944	360944128	762	580644	442450728
713	508369	362467097	763	582169	444194947
714	509796	363994344	764	583696	445943744
715	511225	365525875	765	585225	447697125
716	512656	367061696	766	586756	449455096
717	514089	368601813	767	588289	451217663
718	515524	360146232	768	589824	452984832
719	516961	371694959	769	591361	454756609
720	518400	373248000	770	592900	456533000
721	519841	374805361	771	594441	458314011
722	521284	376367048	772	595984	460099648
723	522729	377933067	773	597529	461889917
724	524176	379503424	774	599076	463684824
725	525625	381078125	775	600625	465484375
726	527076	382657176	776	602176	467288576
727	528529	384240583	777	603729	469097433
728	529984	385828352	778	605284	470910952
729	531441	387420489	779	606841	472729139
730	532900	389017000	780	608400	474552000
731	534361	390617891	781	609961	476379541
732	535824	392223168	782	611524	478211768
733	537289	393832837	783	613089	480048687
734	538756	395446904	784	614656	481890304
735	540225	397065375	785	616225	483736025
736	541696	398688256	786	617796	485587656
737	543169	400315553	787	619369	487443403
738	544644	401947272	788	620944	489303872
739	546121	403583419	789	622521	491169069
740	547600	405224000	790	624100	493039000
741	549081	406869021	791	625681	494913671
742	550564	408518488	792	627264	496793088
743	552049	410172407	793	628849	498677257
744	553536	411830784	794	630436	500566184
745	555025	413493625	795	632025	502459875
746	556516	415160936	796	633616	504358336
747	558009	416832723	797	635209	506261573
748	559504	418508992	798	636804	508169592
749	561001	420189749	799	638401	510082399
750	562500	421875000	800	640000	512000000

Number.	Square.	Cube.	Number.	Square.	Cube.
801	641601	513922401	851	724201	616295051
802	643204	515849608	852	725904	618470208
803	644809	517781627	853	727609	620650477
804	646416	519718464	854	729316	622835864
805	648025	521660125	855	731025	625026375
806	649636	523606616	856	732736	627222016
807	651249	525557943	857	734449	629422793
808	652864	527514112	858	736164	631628712
809	654481	529475129	859	737881	633839779
810	656100	531441000	860	739600	636056000
811	657721	533411731	861	741321	638277381
812	659344	535387328	862	743044	640503928
813	660969	537367797	863	744769	642735647
814	662596	539353144	864	746496	644972544
815	664225	541343375	865	748225	647214625
816	665856	543338496	866	749956	649461896
817	667489	545338513	867	751689	651714363
818	669124	547343432	868	753424	653972032
819	670761	549353259	869	755161	656234909
820	672400	551368000	870	756900	658503000
821	674041	553387661	871	758641	660776311
822	675684	555412248	872	760384	663054848
823	677329	557441767	873	762129	665338617
824	678976	559476224	874	763876	667627624
825	680625	561515625	875	765625	669921875
826	682276	563559976	876	767376	672221376
827	683929	565609283	877	769129	674526133
828	685584	567663552	878	770884	676836152
829	687241	569722789	879	772641	679151439
830	688900	571787000	880	774400	681472000
831	690561	573856191	881	776161	683797841
832	692224	575930368	882	777924	686128968
833	693889	578009537	883	779689	688465387
834	695556	580093704	884	781456	690807104
835	697225	582182875	885	783225	693154125
836	698896	584277056	886	784996	695506456
837	700569	586376253	887	786769	697864103
838	702244	588480472	888	788544	700227072
839	703921	590589719	889	790321	702595369
840	705600	592704000	890	792100	704969000
841	707281	594823321	891	793881	707347971
842	708964	596947688	892	795664	709732288
843	710649	599077107	893	797449	712121957
844	712336	601211584	894	799236	714516984
845	714025	603351125	895	801025	716917375
846	715716	605495736	896	802816	719323136
847	717409	607645423	897	804609	721734273
848	719104	609800192	898	806404	724150792
849	720801	611960049	899	808201	726572699
850	722500	614125000	900	810000	729000000

Number.	Square.	Cube.	Number.	Square.	Cube
901	811801	731432701	951	904401	860085351
902	813604	733870808	952	906304	862801408
903	815409	736314327	953	908209	865523177
904	817216	738763264	954	910116	868250664
905	819025	741217625	955	912025	870983875
906	820836	743677416	956	913936	873722816
907	822649	746142643	957	915849	876467493
908	824464	748613312	958	917764	879217912
909	826281	751089429	959	919684	881974079
910	828100	753571000	960	921600	884736000
911	829921	756058031	961	923521	887503681
912	831744	758550528	962	925444	890277128
913	833569	761048497	963	927369	893056347
914	835396	763551944	964	929296	895841344
915	837225	766060875	965	931225	898632125
916	839056	768575296	966	933156	901428696
917	840889	771095213	967	935089	904231063
918	842724	773620632	968	937024	907039232
919	844561	776151559	969	938961	909853209
920	846400	778688000	970	940900	912673000
921	848241	781229961	971	942841	915498611
922	850084	783777448	972	944784	918330048
923	851929	786330467	973	946729	921167317
924	853776	788889024	974	948676	924010424
925	855625	791453125	975	950625	926859375
926	857476	794022776	976	952576	929714176
927	859329	796597983	977	954529	932574833
928	861184	799178752	978	956484	935441352
929	863041	801765089	979	958441	938313739
930	864900	804357000	980	960400	941192000
931	866761	806954491	981	962361	944076141
932	868624	809557568	982	964324	946966168
933	870489	812166237	983	966289	949862087
934	872356	814780504	984	968256	952763904
935	874225	817400375	985	970225	955671625
936	876096	820025856	986	972196	958585256
937	877969	822656953	987	974169	961504803
938	879844	825293672	988	976144	964430272
939	881721	827936019	989	978121	967361669
940	883600	830584000	990	980100	970299000
941	885481	833237621	991	982081	973242271
942	887364	835896888	992	984064	976191488
943	889249	838561807	993	986049	979146657
944	891136	841232384	994	988036	982107784
945	893025	843908625	995	990025	985074875
946	894916	846590536	996	992016	988047936
947	896809	849278123	997	994009	991026973
948	898704	851971392	998	996004	994011992
949	900601	854670349	999	998001	997002999
950	902500	857375000	1000	1000000	1000000000

To find the Square of a greater Number than is contained in the Table.

RULE 1.—If the number required to be squared exceed, by 2, 3, 4, or any other number of times, any number contained in the table, let the square affixed to the number in the table be multiplied by the square of 2, 3, or 4, &c., and the product will be the answer sought.

EXAMPLE.—Required the square of 2595.

2595 is three times greater than 865; and the square of 865, as per table, is 748225.

Then, $748225 \times 3^2 = 6734025$, Ans.

RULE 2.—If the number required to be squared be an odd number, and do not exceed twice the amount of any number contained in the table, find the two numbers nearest to each other, which, added together, make that sum; then, the sum of the squares of these two numbers, as per table, multiplied by 2, will exceed the square required by 1.

EXAMPLE.—Required the square of 1865.

$$\text{Two nearest numbers } \left.\begin{matrix} 932 \\ 933 \end{matrix}\right\} = 1865.$$

$$\text{Then, per table, } \left\{\begin{matrix} 932^2 = 868624 \\ 933^2 = 870489 \end{matrix}\right\} = 1739113 \times 2 = 3478226 - 1$$
$= 3478225$, Ans.

To find the Cube of a greater Number than is contained in the Table.

RULE.—Proceed, as in squares, to find how many times the number required to be cubed exceeds a number contained in the table. Multiply the cube of that number by the cube of as many times as the number sought exceeds the number in the table, and the product will be the answer required.

EXAMPLE.—Required the cube of 3984.

3984 is 4 times greater than 996; and the cube of 996, as per table, is 988047936.

Then, $988047936 \times 4^3 = 63235067904$, Ans.

To find the Squares of Numbers following each other in arithmetical progression.

RULE.—Find, in the usual manner, the squares of the first two numbers, and subtract the less from the greater. Bring down, in a separate column, the square of the largest of these two numbers, and add it to the difference, with the addition of 2 as a constant quantity; the product will be the square of the next ensuing number.

EXAMPLE 1.—Suppose it be required to extend the foregoing table of squares.

$$\begin{aligned} \text{Then, } 1000^2 &= 1000000 \\ 999^2 &= \underline{998001} \\ & \quad\; 1999 \text{ Difference.} \end{aligned}$$

	1000000	the Square of 1000.
Difference 1999 + 2 =	2001	
	1002001	the Square of 1001.
Difference 2001 + 2 =	2003	
	1004004	the Square of 1002.
Difference 2003 + 2 =	2005	
	1006009	the Square of 1003.
Difference 2005 + 2 =	2007	
	1008016	

In a similar manner the squares of any numbers, following each other in arithmetical progression may be found; or the foregoing table may easily be extended to any required length.

To find the Cubes of Numbers following each other in arithmetical progression.

The cubes of a natural series of numbers may be found by a method very similar to that used for the squares; but as two series of differences have to be added, in the cubes, the operation is somewhat more complex.

Rule.—Find the cubes of the first two numbers, and subtract the less from the greater. Then, multiply the least of the two numbers cubed by 6; add the product, with the addition of 6 as a constant quantity, to the difference; and thus continue the first series of differences.

For the second series of differences, bring down, in a separate column, the cube of the highest of the above numbers, and add the difference to it. The amount will be the cube of the next general number.

Example.—Required the cubes of 1001, 1002, 1003, and 1004.

First series of differences.

Per Tab. 10003 =	1000000000	
9293 =	997002999	
	2997001	diff.
999 x 6 + 6 =	6000	
	3003001	diff. of 1000.
6000 + 6 =	6006	
	3009007	diff. of 1001.
6006 + 6 =	6012	
	3015019	diff. of 1002.
6012 + 6 =	6018	
	3021037	diff. of 1003.

Second series of differences.

Then,	1000000000	Cube of 1000.
Diff. for 1000 =	3003001	
	1003003001	Cube of 1001.
Diff. for 1001 =	3009007	
	1006012008	Cube of 1002.
Diff. for 1002 =	3015019	
	1009027027	Cube of 1003.
Diff for 1003 =	3021037	
	1012048064	Cube of 1004.

TABLES OF SQUARE AND CUBE ROOTS,

To facilitate the Mensuration of the Surfaces and Solidities of Bodies.

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
1	1.0	1.0	50	7.0710678	3.684031
2	1.4142136	1.259921	51	7.1414284	3.708430
3	1.7320508	1.442250	52	7.2111026	3.732511
4	2.0	1.587401	53	7.2801099	3.756286
5	2.2360680	1.709976	54	7.3484692	3.779763
6	2.4494897	1.817121	55	7.4161985	3.802953
7	2.6457513	1.912933	56	7.4833148	3.825862
8	2.8284271	2.0	57	7.5498344	3.848501
9	3.0	2.080084	58	7.6157731	3.870877
10	3.1622777	2.154435	59	7.6811457	3.892996
11	3.3166248	2.223980	60	7.7459667	3.914867
12	3.4641016	2.289428	61	7.8102497	3.936497
13	3.6055513	2.351335	62	7.8740079	3.957892
14	3.7416574	2.410142	63	7.9372539	3.979057
15	3.8729833	2.466212	64	8.0	4.0
16	4.0	2.519842	65	8.0622577	4.020726
17	4.1231056	2.571282	66	8.1240384	4.041240
18	4.2426407	2.620741	67	8.1853528	4.061548
19	4.3588989	2.668402	68	8.2462113	4.081656
20	4.4721360	2.714418	69	8.3066239	4.101566
21	4.5825757	2.758923	70	8.3666003	4.121285
22	4.6904158	2.802039	71	8.4261498	4.140818
23	4.7958315	2.843867	72	8.4852814	4.160168
24	4.8989795	2.884499	73	8.5440037	4.179339
25	5.0	2.924018	74	8.6023253	4.198336
26	5.0990195	2.962496	75	8.6602540	4.217163
27	5.1961524	3.0	76	8.7177979	4.235824
28	5.2915026	3.036589	77	8.7749644	4.254321
29	5.3851648	3.072317	78	8.8317609	4.272659
30	5.4772256	3.107232	79	8.8881944	4.290841
31	5.5677644	3.141381	80	8.9442719	4.308870
32	5.6568542	3.174802	81	9.0	4.326749
33	5.7445626	3.207534	82	9.0553851	4.344481
34	5.8309519	3.239612	83	9.1104336	4.362071
35	5.9160798	3.271066	84	9.1651514	4.379519
36	6.0	3.301927	85	9.2195445	4.396830
37	6.0827625	3.332222	86	9.2736185	4.414005
38	6.1644140	3.361975	87	9.3273791	4.431047
39	6.2449980	3.391211	88	9.3808315	4.447960
40	6.3245553	3.419952	89	9.4339811	4.464745
41	6.4031242	3.448217	90	9.4868330	4.481405
42	6.4807407	3.476027	91	9.5393920	4.497942
43	6.5574385	3.503398	92	9.5916630	4.514357
44	6.6332496	3.530348	93	9.6436508	4.530655
45	6.7082039	3.556893	94	9.6953597	4.546836
46	6.7823300	3.583048	95	9.7467943	4.562903
47	6.8556546	3.608826	96	9.7979590	4.577857
48	6.9283032	3.634241	97	9.8488578	4.594701
49	7.0	3.659306	98	9.8994949	4.610436

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
99	9.9498744	4.626065	150	12.2474487	5.313293
100	10.0	4.641589	151	12.2882057	5.325074
101	10.0498756	4.657010	152	12.3288280	5.336803
102	10.0995049	4.672330	153	12.3693169	5.348481
103	10.1488916	4.687548	154	12.4096736	5.360108
104	10.1980390	4.702669	155	12.4498996	5.371685
105	10.2469508	4.717694	156	12.4899960	5.383231
106	10.2956301	4.732624	157	12.5299641	5.394690
107	10.3440804	4.747459	158	12.5698051	5.406120
108	10.3923048	4.762203	159	12.6095202	5.417501
109	10.4403065	4.776856	160	12.6491106	5.428835
110	10.4880885	4 791420	161	12.6885775	5.440122
111	10.5356538	4.805896	162	12.7279221	5.451362
112	10.5830052	4.820284	163	12.7671453	5.462556
113	10.6301458	4.834588	164	12.8062485	5.473703
114	10.6770783	4.848808	165	12.8452326	5.484806
115	10.7238053	4.862944	166	12.8840987	5.495865
116	10.7703296	4.876999	167	12.9228480	5.506879
117	10.8166538	4.890973	168	12.9614814	5.517848
118	10.8627805	4.904868	169	13.0	5.528775
119	10.9087121	4.918685	170	13.0384048	5.539658
120	10.9544512	4.932424	171	13.0766968	5.550499
121	11.0	4.946088	172	13.1148770	5.561298
122	11.0453610	4 959675	173	13.1529464	5.572054
123	11.0905365	4.973190	174	13.1909060	5.582770
124	11.1355287	4.986631	175	13.2287566	5.593445
125	11.1803399	5.0	176	13.2664992	5.604079
126	11.2249722	5.013298	177	13.3041347	5.614673
127	11.2694277	5.026526	178	13.3416641	5.625226
128	11.3137085	5.039684	179	13.3790882	5.635741
129	11.3578167	5.052774	180	13.4164079	5.646216
130	11.4017543	5.065797	181	13.4536240	5.656652
131	11.4455231	5.078753	182	13.4907376	5.667051
132	11.4891253	5.091643	183	13.5277493	5.677411
133	11.5325626	5.104469	184	13.5646600	5.687734
134	11.5758369	5.117230	185	13.6014705	5.698019
135	11.6189500	5.129928	186	13.6381817	5.708267
136	11.6619038	5.142563	187	13.6747943	5.718479
137	11.7046999	5.155137	188	13.7113092	5.728654
138	11.7473444	5.167649	189	13.7477271	5.738794
139	11.7898261	5.180101	190	13.7840488	5.748897
140	11.8321596	5.192494	191	13.8202750	5.758965
141	11.8743421	5.204828	192	13.8564065	5.768998
142	11.9163753	5.217103	193	13.8924440	5.778996
143	11.9582607	5.229321	194	13.9283883	5.788960
144	12.0	5.241482	195	13.9642400	5.798890
145	12.0415946	5.253588	196	14.0	5.808786
146	12.0830460	5.265637	197	14.0356688	5.818648
147	12.1243557	5.277632	198	14.0712473	5.828476
148	12.1655251	5.289572	199	14.1067360	5.838272
149	12.2065556	5.301459	200	14.1421356	5.848035

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
201	14.1774469	5.857765	251	15.8429795	6.307992
202	14.2126704	5.867464	252	15.8745079	6.316359
203	14.2478068	5.877130	253	15.9059737	6.324704
204	14.2828569	5.886765	254	15.9373775	6.333025
205	14.3178211	5.896368	255	15.9687194	6.341325
206	14.3527001	5.905941	256	16.0	6.349602
207	14.3874946	5.915481	257	16.0312195	6.357859
208	14.4222051	5.924991	258	16.0623784	6.366095
209	14.4568323	5.934473	259	16.0934769	6.374310
210	14.4913767	5.943911	260	16.1245155	6.382504
211	14.5258390	5.953341	261	16.1554944	6.390676
212	14.5602198	5.962731	262	16.1864141	6.398827
213	14.5945195	5.972091	263	16.2172747	6.406958
214	14.6287388	5.981426	264	16.2480768	6.415068
215	14.6628783	5.990727	265	16.2788206	6.423157
216	14.6969385	6.0	266	16.3095064	6.431226
217	14.7309199	6.009244	267	16.3401346	6.439275
218	14.7648231	6.018363	268	16.3707055	6.447305
219	14.7986486	6.027650	269	16.4012195	6.455314
220	14.8323970	6.036811	270	16.4316767	6.463304
221	14.8660687	6.045943	271	16.4620776	6.471274
222	14.8996644	6.055048	272	16.4924225	6.479224
223	14.9331845	6.064126	273	16.5227116	6.487153
224	14.9666295	6.073177	274	16.5529454	6.495064
225	15.0	6.082201	275	16.5831240	6.502956
226	15.0332964	6.091199	276	16.6132477	6.510829
227	15.0665192	6.100170	277	16.6433170	6.518684
228	15.0996689	6.109115	278	16.6733320	6.526519
229	15.1327460	6.118032	279	16.7032931	6.534335
230	15.1657509	6.126925	280	16.7332005	6.542132
231	15.1986842	6.135792	281	16.7630546	6.549911
232	15.2315462	6.144634	282	16.7928556	6.557672
233	15.2643375	6.153449	283	16.8226038	6.565415
234	15.2970585	6.162239	284	16.8522995	6.573139
235	15.3297097	6.171005	285	16.8819430	6.580844
236	15.3622915	6.179747	286	16.9115345	6.588531
237	15.3948043	6.188463	287	16.9410743	6.596202
238	15.4272486	6.197154	288	16.9705627	6.603854
239	15.4596248	6.205821	289	17.0	6.611488
240	15.4919334	6.214464	290	17.0293864	6.619106
241	15.5241747	6.223083	291	17.0587221	6.626705
242	15.5563492	6.231678	292	17.0880075	6.634287
243	15.5884573	6.240251	293	17.1172428	6.641851
244	15.6204994	6.248800	294	17.1464282	6.649399
245	15.6524758	6.257324	295	17.1755640	6.656930
246	15.6843871	6.265826	296	17.2046505	6.664443
247	15.7162336	6.274304	297	17.2336876	6.671940
248	15.7480157	6.282760	298	17.2626765	6.679419
249	15.7797338	6.291194	299	17.2916165	6.686882
250	15.8113883	6.299604	300	17.3205081	6.694328

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
301	17.3493516	6.701758	351	18.7349940	7.054003
302	17.3781472	6.709172	352	18.7616630	7.060696
303	17.4068952	6.716569	353	18.7882942	7.067376
304	17.4355958	6.723950	354	18.8148877	7.074043
305	17.4642492	6.731316	355	18.8414437	7.080698
306	17.4928557	6.738665	356	18.8679623	7.087341
307	17.5214155	6.745997	357	18.8944436	7.093970
308	17.5499288	6.753313	358	18.9208879	7.100588
309	17.5783958	6.760614	359	18.9472953	7.107193
310	17.6068169	6.767899	360	18.9736660	7.113786
311	17.6351921	6.775168	361	19.0	7.120367
312	17.6635217	6.782422	362	19.0262976	7.126935
313	17.6918060	6.789661	363	19.0525589	7.133492
314	17.7200451	6.796884	364	19.0787840	7.140037
315	17.7482393	6.804091	365	19.1049732	7.146569
316	17.7763888	6.811284	366	19.1311265	7.153090
317	17.8044938	6.818461	367	19.1572441	7.159599
318	17.8325545	6.825624	368	19.1833261	7.166095
319	17.8605711	6.832771	369	19.2093727	7.172580
320	17.8885438	6.839903	370	19.2353841	7.179054
321	17.9164729	6.847021	371	19.2613603	7.185516
322	17.9443584	6.854124	372	19.2873015	7.191966
323	17.9722008	6.861211	373	19.3132079	7.198405
324	18.0	6.868284	374	19.3390796	7.204832
325	18.0277564	6.875343	375	19.3649167	7.211247
326	18.0554701	6.882388	376	19.3907194	7.217652
327	18.0831413	6.889419	377	19.4164878	7.224045
328	18.1107703	6.896435	378	19.4422221	7.230427
329	18.1383571	6.903436	379	19.4679223	7.236797
330	18.1659021	6.910423	380	19.4935887	7.243156
331	18.1934054	6.917396	381	19.5192213	7.249504
332	18.2208672	6.924355	382	19.5448203	7.255841
333	18.2482876	6.931300	383	19.5703858	7.262167
334	18.2756669	6.938232	384	19.5959179	7.268482
335	18.3030052	6.945149	385	19.6214169	7.274786
336	18.3303028	6.952053	386	19.6468827	7.281079
337	18.3575598	6.958943	387	19.6723156	7.287362
338	18.3847763	6.965819	388	19.6977156	7.293633
339	18.4119526	6.972682	389	19.7230829	7.299893
340	18.4390889	6.979532	390	19.7484177	7.306143
341	18.4661853	6.986369	391	19.7737199	7.312383
342	18.4932420	6.993191	392	19.7989899	7.318611
343	18.5202592	7.0	393	19.8242276	7.324829
344	18.5472370	7.006796	394	19.8494332	7.331037
345	18.5741756	7.013579	395	19.8746069	7.337234
346	18.6010752	7.020349	396	19.8997487	7.343420
347	18.6279360	7.027106	397	19.9248588	7.349596
348	18.6547581	7.033850	398	19.9499373	7.355762
349	18.6815417	7.040581	399	19.9749844	7.361917
350	18.7082869	7.047208	400	20.0	7.368063

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
401	20.0249844	7.374198	451	21.2367606	7.668766
402	20.0499377	7.380322	452	21.2602916	7.674430
403	20.0748599	7.386437	453	21.2837967	7.680085
404	20.0997512	7.392542	454	21.3072758	7.685732
405	20.1246118	7.398636	455	21.8307290	7.691371
406	20.1494417	7.404720	456	21.3541565	7.697002
407	20.1742410	7.410794	457	21.3775583	7.702624
408	20.1990099	7.416859	458	21.4009346	7.708238
409	20.2237484	7.422914	459	21.4242853	7.713844
410	20.2484567	7.428958	460	21.4476106	7.719442
411	20.2731349	7.434993	461	21.4709106	7.725032
412	20.2977831	7.441018	462	21.4941853	7.730614
413	20.3224014	7.447033	463	21.5174348	7.736187
414	20.3469809	7·453039	464	21.5406592	7.741753
415	20.3715488	7.459036	465	21.5638587	7.747310
416	20.3960781	7.465022	466	21.5870331	7.752860
417	20.4205779	7.470999	467	21.6101828	7.758402
418	20.4450483	7.476966	468	21.6333077	7.763936
419	20.4694895	7.482924	469	21.6564078	7.769462
420	20.4939015	7.488872	470	21.6794834	7.774980
421	20.5182845	7.494810	471	21.7025344	7.780490
422	20.5426386	7.500740	472	21.7255610	7.785992
423	20.5669638	7.506660	473	21.7485632	7.791487
424	20.5912603	7.512571	474	21.7715411	7.796974
425	20.6155281	7.518473	475	21.7944947	7.802453
426	20.6397674	7.524365	476	21.8174242	7.807925
427	20.6639783	7.530248	477	21.8403297	7.813389
428	20.6881609	7.536121	478	21.8632111	7.818845
429	20.7123152	7.541986	479	21.8860686	7 824294
430	20.7364414	7.547841	480	21.9089023	7.829735
431	20.7605395	7.553688	481	21.9317122	7.835168
432	20.7846097	7.559525	482	21.9544984	7.840594
433	20.8086520	7.565353	483	21.9772610	7.846013
434	20.8326667	7.571173	484	22.0	7.851424
435	20.8566536	7.576984	485	22.0227155	7.856828
436	20.8806130	7.582786	486	22.0454077	7.862224
437	20.9045450	7 588579	487	22.0680765	7.867613
438	20.9284495	7.594363	488	22.0907220	7.872994
439	20.9523268	7.600138	489	22.1133444	7.878368
440	20.9761770	7.605905	490	22.1359436	7.883734
441	21.0	7.611662	491	22.1585198	7.889094
442	21.0237960	7.617411	492	22.1810730	7.894446
443	21.0475652	7.623151	493	22.2036033	7.899791
444	21.0713075	7.628883	494	22 2261108	7.905129
445	21.0950231	7.634606	495	22.2485955	7.910460
446	21.1187121	7.640321	496	22.2710575	7.915784
447	21.1423745	7.646027	497	22.2934968	7.921100
448	21.1660105	7.651725	498	22.3159136	7.926408
449	21.1896201	7.657415	499	22.3383079	7.931710
450	21.2132044	7.663094	500	22.3606798	7.937005

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
501	22.3830293	7.942293	551	23.4733892	8.198175
502	22.4053565	7.947573	552	23.4946802	8.203131
503	22.4276615	7.952847	553	23.5159520	8.208082
504	22.4499443	7.958114	554	23.5372046	8.213027
505	22.4722051	7.963374	555	23.5584380	8.217965
506	22.4944438	7.968627	556	23.5796522	8.222898
507	22.5166605	7.973873	557	23.6008474	8.227825
508	22 5388553	7.979112	558	23.6220236	8.232746
509	22.5610283	7.984344	559	23 6431808	8.237661
510	22.5831796	7.989569	560	23 6643191	8.242570
511	22.6053091	7.994788	561	23.6854386	8.247474
512	22.6274170	8 0	562	23.7065392	8.222371
513	22.6495033	8.005205	563	23.7276210	8.257263
514	22.6715681	8.010403	564	23.7486842	8.262149
515	22.6936114	8.015595	565	23.7697286	8.267029
516	22.7156334	8.020779	566	23.7907545	8.271903
517	22.7376340	8.025957	567	23.8117618	8.276772
518	22.7596134	8.031120	568	23.8327506	8.281635
519	22.7815715	8.036293	569	23.8537209	8.286493
520	22.8035085	8.041451	570	23.8746728	8.291344
521	22.8254244	8.046603	571	23 8956063	8.296190
522	22.8473193	8.051748	572	23.9165215	8.301030
523	22.8691933	8.056886	573	23.9374184	8.305865
524	22.8910463	8.062018	574	23.9582971	8 310694
525	22.9128785	8.067143	575	23.9791576	8.315517
526	22.9346899	8.072262	576	24.0	8.320335
527	22.9564806	8.077374	577	24.0208243	8.325147
528	22.9782506	8.082480	578	24.0416306	8.329954
529	23.0	8.087579	579	24.0624188	8.334755
530	23.0217289	8.092672	580	24.0831892	8.339551
531	23.0434372	8.097758	581	24.1039416	8.344341
532	23.0651252	8.102838	582	24.1246762	8.349125
533	23.0867928	8.107912	583	24.1453929	8.353904
534	23.1084400	8.112980	584	24.1660919	8.358678
535	23.1300670	8.118041	585	24.1867732	8.363446
536	23.1516738	8.123096	586	24.2074369	8.368209
537	23.1732605	8.128144	587	24.2280829	8.372966
538	23.1948270	8.133186	588	24.2487113	8.377718
539	23.2163735	8.138223	589	24.2693222	8.382465
540	23.2379001	8.143253	590	24.2899156	8.387206
541	23.2594067	8.148276	591	24.3104916	8.391942
542	23.2808935	8.153293	592	24.3310501	8.396673
543	23.3023604	8.158304	593	24.3515913	8.401398
544	23.3238076	8.163309	594	24.3721152	8.406118
545	23.3452351	8.168308	595	24.3926218	8.410832
546	23.3666429	8.173302	596	24.4131112	8.415541
547	23.3880311	8.178289	597	24.4335832	8.420245
548	23.4093998	8.183269	598	24.4540385	8.424944
549	23.4307490	8.188244	599	24.4744765	8.429638
550	23.4520788	8.193212	600	24.4948974	8.434327

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
601	24.5153013	8.439009	651	25.5147016	8.666831
602	24 5356883	8.443687	652	25.5342907	8.671266
603	24.5560583	8 448360	653	25.5538647	8.675697
604	24 5764115	8.453027	654	25.5734237	8.680123
605	24.5967478	8.457689	655	25.5929678	8.684545
606	24 6170673	8.462347	656	25.6124969	8.688963
607	24.6373700	8.466999	657	25.6320112	8.693376
608	24 6576560	8.471647	658	25.6515107	8.697784
609	24.6779254	8.476289	659	25.6709953	8.702188
610	24.6981781	8.480926	660	25.6904652	8.706587
611	24.7184142	8.485557	661	25.7099203	8.710982
612	24.7386338	8.490184	662	25.7203607	8.715373
613	24.7588368	8.494806	663	25.7487864	8.719759
614	24.7790234	8.499423	664	25.7681975	8.724141
615	24.7991935	8.504034	665	25.7875939	8.728518
616	24.8193473	8.508641	666	25.8069758	8.732891
617	24.8394847	8.513243	667	25.8263431	8.737260
618	24.8596058	8.517840	668	25.8456960	8.741624
619	24.8797106	8.522432	669	25.8650343	8.745984
620	24.8997992	8.527018	670	25.8843582	8.750340
621	24.9198716	8.531600	671	25.9036677	8.754691
622	24.9399278	8.536177	672	25.9229628	8.759038
623	24 9599679	8.540749	673	25.9422435	8.763380
624	24.9799920	8.545317	674	25.9615100	8.767719
625	25 0	8.549879	675	25.9807621	8.772053
626	25.0199920	8.554437	676	26.0	8.776382
627	25.0399681	8.558990	677	26.0192237	8.780708
628	25.0599282	8.563537	678	26.0384331	8.785029
629	25.0798724	8.568080	679	26.0576284	8.789346
630	25.0998008	8 572618	680	26.0768096	8.793659
631	25.1197134	8.577152	681	26.0959767	8.797967
632	25 1396102	8.581680	682	26.1151297	8.802272
633	25.1594913	8.586204	683	26.1342687	8.806572
634	25.1793566	8.590723	684	26.1533937	8.810868
635	25.1992063	8.595238	685	26.1725047	8.815159
636	25.2190404	8.599747	686	26.1916017	8.819447
637	25.2388589	8.604252	687	26.2106848	8.823730
638	25.2586619	3.608752	688	26.2297541	8.828009
639	25.2784493	8.613248	689	26.2488095	8.832285
640	25.2982213	8.617738	690	26.2678511	8.836556
641	25.3179778	8.622224	691	26.2868789	8.840822
642	25.3377189	8.626706	692	26.3058929	8.845085
643	25.3574447	8.631183	693	26.3248932	8.849344
644	25.3771551	8.635655	694	26.2438797	8.853598
645	25.3968502	8.640122	695	26.3628527	8.857849
646	25.4165301	8.644585	696	26.3818119	8.862095
647	25.4361917	8.649043	697	26.4007576	8 866337
648	25.4558441	8.653497	698	26.4196896	8.870575
649	25.4754784	8.657946	699	26.4386081	8.874809
650	25.4950076	8.662301	700	26.4575131	8.879040

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
701	26.4764046	8.883266	751	27.4043792	9.089639
702	26.4952826	8.887488	752	27.4226184	9.093672
703	26.5141472	8.891706	753	27.4408455	9.097701
704	26.5329983	8.895920	754	27.4590604	9.101726
705	26.5518361	8.900130	755	27.4772633	9.105748
706	26.5706605	8.904336	756	27.4954542	9.109766
707	26.5894716	8.908538	757	27.5136330	9.113781
708	26.6082694	8.912736	758	27.5317998	9.117793
709	26.6270539	8.916931	759	27.5499546	9.121801
710	26.6458252	8.921121	760	27.5680975	9.125805
711	26.6645833	8.925307	761	27.5862284	9.129806
712	26.6833281	8.929490	762	27.6043475	9.133803
713	26.7020598	8 933668	763	27.6224546	9.137797
714	26.7207784	8.937843	764	27.6405499	9.141788
715	26.7394839	8.942014	765	27.6586334	9.145774
716	26.7581763	8.946180	766	27.6767050	9.149757
717	26.7768557	8 950343	767	27.6947648	9.153737
718	26.7955220	8.954502	768	27.7128129	9.157713
719	26.8141754	8.958658	769	27.7308492	9.161686
720	26.8328157	8.962809	770	27.7488739	9.165656
721	26.8544432	8.966957	771	27.7668868	9.169622
722	26.8700577	8.971100	772	27.7848880	9.173585
723	26.8886593	8.975240	773	27.8028775	9.177544
724	26.9072481	8.979376	774	27.8208555	9.181500
725	26.9258240	8.983508	775	27.8388218	9.185452
726	26.9444872	8.987637	776	27.8567766	9.189401
727	26.9629375	8.991762	777	27.8747197	9.193347
728	26.9814751	8.995883	778	27.8926514	9.197289
729	27.0	9.0	779	27.9105715	9.201228
730	27.0185122	9.004113	780	27.9284801	9.205164
731	27.0370117	9.008222	781	27.9463772	9.209096
732	27.0554985	9.012328	782	27.9642629	9.213025
733	27.0739727	9.016430	783	27.9821372	9.216950
734	27.0924344	9.020529	784	28.0	9.220872
735	27.1108834	9.024623	785	28.0178515	9.224791
736	27.1293199	9.028714	786	28.0356915	9.228706
737	27.1477439	9.032802	787	28.0535203	9.232618
738	27.1661554	9.036885	788	28.0713377	9.237527
739	27.1845544	9.040965	789	28.0891438	9.240433
740	27.2029410	9.045041	790	28.1069386	9.244335
741	27.2213152	9.049114	791	28.1247222	9.248234
742	27.2396769	9.053183	792	28.1424946	9.252130
743	27.2580263	9.057248	793	28.1602557	9.256022
744	27.2763634	9.061309	794	28.1780056	9.259911
745	27.2946881	9.065367	795	28.1957444	9.263797
746	27.3130006	9.069422	796	28.2134720	9.267679
747	27.3313007	9 073472	797	28.2311884	9.271559
748	27.3495887	9.077519	798	28.2488938	9.275435
749	27.3678644	9.081563	799	28.2665881	9.279308
750	27.3861279	9.085603	800	28.2842712	9.283177

Number.	Square Root	Cube Root.	Number.	Square Root.	Cube Root.
801	28.3019434	9.287044	851	29.1719013	9.476395
802	28.3196045	9.290907	852	29.1890390	9.480106
803	28.3372546	9.294767	853	29.2061637	9.483813
804	28.3548938	9.298623	854	29.2232784	9.487518
805	28.3725219	9.302477	855	29.2403830	9.491219
806	28.3901391	9.306327	856	29.2574777	9.494918
807	28.4077454	9.310175	857	29.2745623	9.498614
808	28.4253408	9.314019	858	29.2916370	9.502307
809	28.4429253	9.317859	859	29.3087018	9.505998
810	28.4604989	9.321697	860	29.3257566	9.509685
811	28.4780617	9.325532	861	29.3428015	9.513369
812	28.4956137	9.329363	862	29.3598365	9.517051
813	28.5131549	9.333191	863	29.3768616	9.520730
814	28.5306852	9.337016	864	29.3938769	9.524406
815	28.5482048	9.340838	865	29.4108823	9.528079
816	28.5657137	9.344657	866	29.4278779	9.531749
817	28.5832119	9.348473	867	29.4448637	9.535417
818	28.6006993	9.352285	868	29.4618397	9.539081
819	28.6181760	9.356095	869	29.4788059	9 542743
820	28.6356421	9.359901	870	29.4957624	9.546402
821	28.6530976	9.363704	871	29.5127091	9.550058
822	28.6705424	9.367505	872	29.5296461	9.553712
823	22.6879766	9.371302	873	29.5465734	9.557363
824	28.7054002	9.375096	874	29.5634910	9.561010
825	28.7228132	9.378887	875	29.5803989	9.564655
826	28.7402157	9.372675	876	29.5972972	9.568297
827	28.7576077	9.386460	877	29.6141858	9.571937
828	28.7749891	9 390241	878	29,6310648	9.575574
829	28.7923601	9.394020	979	29.6479325	9.579208
830	28.8097206	9.397796	880	29 6647939	9.582839
831	28.8270706	9.401569	881	29.6816442	9.586468
832	28.8444102	9.405338	882	29.6984848	9.590093
833	28.8617394	4.409105	883	29.7153159	9.593716
834	28.8790582	9.412869	884	29.7321375	9.597337
835	28.8963666	9.416630	885	29.7489496	9.600954
836	28.9136646	9.420387	886	29.7557521	9.604569
837	28.9309523	9.424141	887	29.7825452	9.608181
838	28.9482297	9.427893	888	29.7993289	9.611791
839	28.9654967	9.431642	889	29.8161030	9.615397
840	28 9827535	9.435388	890	29.8328678	9.619001
841	29.0	9.439130	891	29.8496231	9.622603
842	29 0172363	9.442870	892	29.8663690	9.626201
843	29.0344623	9.446607	893	29.8831056	9.629797
844	29.0516781	9.450341	894	29.8798328	9.633390
845	29.0688837	9.454071	895	29.9165506	9.636981
846	29 0860791	9.457799	896	29.9332591	9.640569
847	29.1032644	9.461524	897	29.9499583	9.644154
848	29.1204396	9.465247	898	29.9666481	9 647736
849	29.1376046	9.468966	899	29.9833287	9.651316
850	29 1547595	9.472682	900	30.0	9.654893

Number.	Square Root.	Cube Root.	Number.	Square Root.	Cube Root.
901	30.0166620	9.658468	951	30.8382879	9.833923
902	30.0333148	9.662040	952	30.8544972	9.837369
903	30.0499584	9.665609	953	30.8706981	9.840812
904	30.0665928	9.669176	954	30.8868904	9.844253
905	30.0832179	9.672740	955	30.9030743	9.847692
906	30.0998339	9.676301	956	30.9192497	9.851128
907	30.1164407	9.679860	957	30.9354166	9.854561
908	30.1330383	9.683416	958	30.9515751	9.857992
909	30.1496269	9.686970	959	30.9677251	9.861421
910	30.1662063	9.690521	960	30.9838668	9.864848
911	30.1827765	9.694069	961	31.0	9.868272
912	30.1993377	9.697615	962	31.0161248	9.871694
913	30.2158899	9.701158	963	31.0322413	9.875113
914	30.2324329	[illegible]	964	31.0483494	9.878530
915	30.2489669	[illegible]	965	31.0644491	9.881945
916	30.2654919	9.711772	966	31.0805405	9.885357
917	30.2820079	9.715305	967	31.0966236	9.888767
918	30.2985148	9.718835	968	31.1126984	9.892174
919	30.3150128	9.722363	969	31.1287648	9.895580
920	[illegible]	[illegible]	970	31.1448230	9.898983
921	[illegible]	[illegible]	971	31.1608729	9.902383
922	[illegible]	[illegible]	972	31.1769145	9.905781
923	[illegible]	[illegible]	973	31.1929479	9.909177
924	[illegible]	[illegible]	974	31.2089731	9.912571
925	[illegible]	[illegible]	975	31.2249900	9.915962
926	[illegible]	[illegible]	976	31.2409987	9.919351
927	[illegible]	[illegible]	977	31.2569992	9.922738
928	[illegible]	[illegible]	978	31.2729915	9.926122
929	[illegible]	[illegible]	979	31.2889757	9.929504
930	[illegible]	[illegible]	980	31.3049517	9.932883
931	[illegible]	[illegible]	981	31.3209195	9.936261
932	[illegible]	[illegible]	982	31.3368792	9.939636
933	[illegible]	[illegible]	983	31.3528308	9.943009
934	[illegible]	[illegible]	984	31.3687743	9.946379
935	[illegible]	[illegible]	985	31.3847097	9.949747
936	[illegible]	[illegible]	986	31.4006369	9.953113
937	[illegible]	[illegible]	987	31.4165561	9.956477
938	[illegible]	[illegible]	988	31.4324673	9.959839
939	[illegible]	[illegible]	989	31.4483704	9.963198
940	[illegible]	[illegible]	990	31.4642654	9.966554
941	[illegible]	[illegible]	991	31.4801525	9.969909
942	[illegible]	[illegible]	992	31.4960315	9.973262
943	[illegible]	[illegible]	993	31.5119025	9.976612
944	[illegible]	[illegible]	994	31.5277655	9.979959
945	[illegible]	[illegible]	995	31.5436206	9.983304
946	[illegible]	[illegible]	996	[illegible]	9.986648
947	[illegible]	[illegible]	997	[illegible]	9.989990
948	[illegible]	[illegible]	998	31.5911380	9.993328
949	[illegible]	[illegible]	999	31.6069613	9.996665
950	[illegible]	[illegible]	1000	[illegible]	10.0

To find the cube or square root of a higher number than is contained in the Table.

Rule.—Refer to the Table, and seek in the column of squares or cubes, the number nearest to that number whose root is sought, and the number from which that square or cube is derived will be the answer required, when decimals are not of importance.

Example 1.—Required the square root of 542869; and the number from which that square has been obtained is 737.

Therefore, $\sqrt{542869}=737$ nearly, Ans.

Example 2.—Required the cube root of 419684381.

In the Table of Cubes, the nearest number is 420189749, and the number opposite is 749.

Therefore, $\sqrt[3]{419684381}=749$ nearly, Ans.

To find more correctly the cube root of a higher number than is contained in the Table.

Rule.—Ascertain, by the Table, the nearest cube number to the number given, and call it the assumed cube.

Multiply the assumed cube, and the given number, respectively, by 2; to the product of the assumed cube add the given number, and to the product of the given number add the assumed cube.

Then, by proportion, as the sum of the assumed cube is to the sum of the given number, so is the root of the assumed cube to the root of the given number.

Example.—Required the cube root of 412568555.

Per Table, the nearest number is 411830784; and its cube root is 744.

Therefore, $411830784\times2+412568555=1236230123$.

And, $412568555\times2+411830784=1236967894$.

Hence, as 1236230123 : 1236967894 : : 744 : 744.369 very nearly **Ans.**

To find the Sixth Root of a Number.

Rule.—Take the cube root of its square root.

Example.—What is the $\sqrt[6]{\ }$ of 441?

$\sqrt{441}=21$, and $\sqrt[3]{21}=2.7589$, *Ans.*

ERRATA.

Page 16, 12th line, for "Multiply by" read *Multiplied by.*

" 50, Ex. 6, for "Ans. 100.9445," read 160.9465.

" 63, the cut for the *Prismoid* is imperfect, the line rising from the letter C, should have joined the upper corner of the figure.

Page 112, First line in 1st series of differences, for "10003" read $\overline{1000}^{3}$ and in the line below, for "9293" read $\overline{999}^{3}$.